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MINISTRY OF SUPPLY

SUPERSONIC JET DEFLECTION

Part II. Deflection by Inclined Tubular Extensions

BY

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TABLE OF SYMBOLS

General

p	=	fluid (gauge) pressure	-	$lbs./ins^2$
F	=	force	-	lbs.
M	=	moment	-	lbs. ft.
		or Mach number		
u	=	velocity	-	ft./sec.
θ	=	angle a deflecting member makes with the normal axis		
ϕ	=	angle the reaction of a deflected jet makes with the normal axis		
t	=	nozzle assembly type		
		or time	-	secs.
m	=	mass flow rate	-	lbs./sec.
T	=	temperature	-	$^{\circ}K$ or $^{\circ}C$
A	=	cross sectional area	-	$ft.^2$
γ	=	ratio of specific heats		
s	=	extension tube length	-	ins.

Suffixes

o refers to reservoir

e refers to nozzle exit conditions

t refers to nozzle throat conditions

s refers to side thrust

D refers to direct thrust

c refers to control force

a, b refers to nozzle types 'a' and 'b'

R refers to resultant

Statistical Symbols

Small letters a, b, c etc.	-	tests or results of tests at the different levels of the various factors
Suffixes, o, 1, 2 etc.	-	levels of the factors
Capital letters, A, B, C etc.	-	factors or effects of factors
Round brackets, ( )	-	mean effects, or Operators
Square brackets [ ]	-	total effects

<u>Note</u> for brevity, $(A_0B_0C_1)$ will be denoted by (C_1) etc.

ν = degrees of freedom (d/F)

d = divisor

F = variance ratio

t = Student's ratio

σ = std. deviation of population

s = std. deviation of sample

Σ = sum of

Bar, $\bar{}$ = mean of

$V_{(\ )}$ = error variance of

n = number of observations

ξ = orthogonal polynomial, ξ_1 ξ_2 etc. $\xi_1(\theta)$ $\xi_2(s)$ etc.

v = coefficient of variation - $\left(\dfrac{\text{Standard Error}}{\text{Mean observation}}\right)$ %

SUMMARY

1. Air at 400 to 1,000 lbs./ins.^{2} has been discharged through a model nozzle of $\frac{1}{2}$" throat to produce a jet of Mach number approximately 3.0. Provision has been made to record the direct reaction, lateral thrust and control moment associated with the deflection of this supersonic air jet.

2. A 'tubular extension' method of jet deflection (see fig.5) has been investigated over a wide range of conditions including tube lengths up to approximately 4 x nozzle throat diameter and inclined up to 20° from the normal axis. Two types have been investigated.

3. The system is found to produce stable and reproducible lateral thrusts by a mechanism which is explained and illustrated by shadowgraphs. Judged by the magnitude of the forces, the method is superior to any known method of deflection so far reported.

4. Lateral thrusts of as much as 50% of the direct thrust may be obtained and control moments of the order of 8 lbs. ft. per 100 lbs. lateral thrust are found to be necessary. Most of the data are summarised in figures 20 to 26.

5. Ignoring the many non-linear relationships and considering grand average values, the information can be summarised as follows:-

FORCES

per degree deflection

per 100 lbs. direct thrust

Deflecting Tube Length, S.	Side Thrust, F_S lbs. (little difference between types.)	Control Moment, M_C lbs. ft. (type b about 15% less.)
3.75x throat diam.	2.42	0.16
2.50x " "	2.53	0.15
1.25x " "	1.95	0.08

The direct thrust is reduced by about $\frac{1}{2}$% per degree deflection.

6. The longer tube lengths incline the resultant line of thrust by about 5° more than the tube angle.

7. The optimum tube length for maximum side thrust is found to lie well within the range studied and averages about 2.7 x the throat diameter.

8. The various forces measured vary almost linearly with air reservoir pressure.

9. Hitherto, experiments in this field have been of an ad hoc nature. However, the experiments reported here are intended to constitute a systematic survey of a section of the subject and were thus planned and the results analysed according to sound statistical principles.(Ref.5). The observations of each test carried out are stated and the full analysis of all the independant variables are given. As a result, the precision and reliability of the data found has been ascertained. The information is presented in the form of equations (and graphs and data derived from these) which are useful for design purposes as well as in the form of comparative effects of variables useful for research. A considerable intricity of the results in the universe considered has been revealed and thus the thorough design and analysis have been well justified.

INTRODUCTION

As a result of difficulties experienced in the development of high angle launch aircraft and rocket propulsion in general, this laboratory has undertaken to investigate the forces involved and the effects produced when supersonic gas jets are deflected in various ways.

Little data are available on this topic and so apparatus and experimental methods have been developed which will allow many different methods of jet deflection to be studied. The tubular extension method of deflection, which is the subject of this report, developed from a pair of paddle vanes.

Early investigations (Ref.4) convinced the experimenters that paddle vanes should be curved to follow the jet contour and this opinion led to the consideration of the limiting case where the curved vanes meet to form an envelope around the jet, out of which arose this present investigation. This development is discussed in Part I of this series (Ref.1).

Apart from its practical utility, this method lends itself admirably to pilot experiments which may be directed towards the development of a suitable form of experimentation.

THE APPARATUS

DESCRIPTION

This is illustrated diagrammatically in Fig.1 and in more detail in Fig.2.

A reservoir of 8 ft.^{3} capacity could be filled with clean and fairly dry air compressed up to 120 atmospheres pressure (1,760 lbs./ins.^{2}) and this rapidly discharged via a near-frictionless right-angled pivot and a direct reaction balance, through a $\frac{1}{2}$" model nozzle. Deflection of the jet tends to rotate the system about the pivot but such motion is resisted hydraulically the pressure thus being proportional to the lateral force. Direct reaction and control moment are also measured hydraulically, and these pressures, together with the air reservoir pressure, recorded photographically.

The Pivot Bearing.

This is illustrated in Fig.3. A stout hollow spindle was mounted near its ends and thereby rigidly secured to a solid base plate. The outer diameter of the spindle was hardened and lap finished to form a bearing surface with a robust cylinder. This cylinder had an annular chamber machined from its interior which connects through radial holes with the hollow interior of the spindle. A tapped hole from the exterior of the cylinder communicated with the annular chamber and served as a mounting for the foot of the direct reaction balance. Essential dimensions and constructional details can be seen in the figure.

The assembly thus constitutes a free pivotting elbow joint. The components were designed so that they may not distort under a high interior pressure load and the bearing surfaces, as well as having a very small friction torque, were finished so that air leakage was negligible. Petroleum jelly was found to be the most suitable lubricant.

The Direct Reaction Balance.

This is described in more detail by Coulter (Ref.2) and illustrated in Fig.1 and Fig.2.

A stout metal cylinder 'a' is rigidly mounted vertically (in the pivot bearing cylinder) and contains a well-fitting piston 'b' having upper and lower guides. This piston is drilled axially to form a flow channel and the upper (hollow) guide rod carries the nozzle. The lower annular chamber 'c' in the cylinder is oil filled and supports the weight of the piston and nozzle. This oil chamber is filled through a simple barrel tap and also communicates with a Bourdon tube pressure gauge. The upper guide rod is suitably drilled so that air in the flow channel may freely pass into the upper annular chamber in the cylinder 'd'. Air pressure equal to that in the reservoir thus acts downwards on the upper surface of the piston. The area of this piston is so designed that this pressure force compensates for the upward force set up by air pressure acting against the under side of the guide rod and nozzle shoulder.

In order that the air under pressure may not leak into the oil chamber, long leakage paths are employed and blood holes, 'e' open to the atmosphere.

Castor oil was found to be a suitable lubricant and hydraulic medium.

The Hydraulic Systems.

Those which were used to record control moment and lateral thrust are illustrated by Fig.4.

Metal bellows enveloped a well fitting piston and cylinder pair (Diesel fuel injection pump inserts) which acted as a locating device.

The bellows isolated the interior which could thus be entirely filled with oil. A Bourdon tube gauge recorded the interior pressure and the upper part of the bellows supported the appropriate lever arm via a ball race. Movement under load was considered negligible.

The Nozzle and Deflecting Assembly.

The nozzles were made of brass, important dimensions are given in Fig.5 and flow characteristics are shown in Appendix I. The interiors were very well finished and accurately gauged. They were rigidly attached to the upper part of the direct reaction balance by a steel body which had provision for holding the deflecting device.

The deflecting members were mounted above the nozzle in ball races and carried a rigid lever arm to bear against the hydraulic bellows. Suitable scales were mounted to indicate the position of the deflector accurately.

Fig.5 illustrates the design employed for the two types of tubular deflectors which were used. The design of a swivelling extension presents many problems of detail and a number of slightly different schemes were considered. Type 'b' was chosen as the simplest possible construction and type 'a' as a simple and obvious modification. The study of two types was intended to determine whether such arbitrary features of design had a marked influence on performance.

THE TECHNIQUE OF EXPERIMENTATION

The discharge of a volume of air through the nozzle and a record of the gauge readings constitutes what is referred to as a 'test'.

Preparation for a test involved charging the reservoir to the required pressure (generally 120 ats.), loading and setting the camera, adjusting the deflecting member to the required position and flushing and filling the direct reaction balance oil annulus. Zero readings of the gauges and the air reservoir temperature were noted.

An assistant operator opened the air valve quickly and steadily and photographs were taken as required by operating an electric push button controlling the camera. Photos were generally taken at 100 lbs./ins.^{2} intervals of air reservoir pressure beginning at 1000 lbs./ins.^{2} and ending at 400 lbs./ins.^{2} This selection was at the judgement of the camera operator.

Readings were taken direct from photographic negatives and at the same time the deflection setting was checked. Plate 6 is a sample recording. The gauges record the hydraulic and air reservoir pressures. The photograph shows deflector type 'a' set at $\theta = 10^o$.

Oil gauge readings were then plotted against the air pressure readings for a test as a check against gross errors in recording and readings of the oil gauges interpolated to the pressure level required, whenever the photograph had not been taken at exactly the right air pressure.

Such figures are regarded as the fundamental observations (as recorded, for example, in Table 8).

THE TESTING AND CALIBRATION OF THE APPARATUS

Testing.

Because of the short action time of a test (about 10 seconds) and the steadily changing conditions, it was considered a first essential to establish that the gauges were adequately responsive and stable. All Bourdon Tubes were evacuated and filled with hydraulic oil which effectively damped any troublesome oscillations. The direct reaction gauge was oil-filled and isolated from the balance by a diaphragm seal. (See Fig.7).

This was mounted close to the balance and was fitted with a bleed screw on the balance side by means of which the hydraulic system could be flushed at each filling. In this way it was ensured that all air was removed.

By controlling the rate at which the air valve was opened, the rate of increase of air pressure could be varied and in this way it was shown that the results were independant of the rate of change of reservoir pressure (over the range considered).

It was further established that dynamic equilibrium prevailed during a test by comparing observations made with rising and falling reservoir pressure. These revealed some small hysteresis but it was very similar to that observed during a static calibration and attributed to friction at the bearing surfaces. The direct reaction balance proved particularly troublesome in this respect until castor oil was used as the hydraulic medium. The excellent lubricating properties of this oil reduced the hysteresis effect to within tolerable limits.

An extensive experimental programme of some 16 tests (known as Experiment I) was conducted to verify that reproducible results were obtained under a variety of operating conditions. As a result of these, it was established that suitable precision could be obtained especially if the operating conditions were standardised. Under these conditions the direct reaction (the accurate measurement of which had proved the most difficult) was found to have a standard deviation of ± 3.2 lbs. force (20 d/F). This is approximately ± 1.5% of the average value recorded.

Calibration.

The direct reaction gauge and diaphragm seal assembly was calibrated on a dead weight tester and reaction calculated from the dimensions of the balance (See Coulter Ref.2).

This leads to,

$$F_D = 3.149\,(p_D - 10.4) \text{ lbs. force}$$

(10.4 lbs./ins.^{2} is the zero gauge reading due to the dead weight of the floated part of the apparatus)

It should be noted that this expression ignores the small thrust generated by the gas acceleration at the foot of the balance. The correction is only necessary when it is required to calculate absolute efficiencies. (See Coulter Ref.2).

The 0 - 1,000 lbs./ins.^{2} air pressure gauge was compared with a sub-standard (recently checked by the makers) and found to read 1% high. It was free of hysteresis and could be conveniently read to ± 3 lbs./ins.^{2}

Control moment and side thrust measurements were calibrated by suspending weights from the appropriate levers suitably extended to afford a convenient mechanical advantage. Calibrations were carried out with rising and falling loads, with loading to different maxima and at different angular settings of the levers. Hysteresis between rising and falling loading was found and this attributed to friction at the various bearing surfaces. However, results were consistent for either mode of loading and so it was decided to accept those for falling load this being the condition prevailing during a test.

A regression on the data leads to the following calibration equations,

$$M_c = .201\, p_c - .49 \pm .09 \; (36 \text{ d/F}) \text{ lbs. ft.}$$

$$F_s = .525\, p_s - 5.77 \pm .80 \; (32 \text{ d/F}) \text{ lbs.}$$

(Side thrust, F_s is assumed to act at the nozzle throat and perpendicularly to the axis of normal flow).

The extension tube lengths were as follows,

S_{a_1} = 5/8" + .003". S_{a_2} = 5/8" + .001"

S_{b_1} = 5/8" - .008". S_{b_2} = 5/8" - .002"

Measurements were made with a micrometer.

The nozzle throat dimensions were,

t_a = .5012" diam. $\equiv$ .001369 ft.^{2} c/s. area.

t_b = .4995" diam. $\equiv$ .001360 ft.^{2} c/s. area.

These were determined both by "go" and "no go" ball gauges and by a .003" in 5" taper mandrel and micrometer.

The deflection angle was measured on a draughtsman's protractor mounted on the lever. The angle changed very slightly under load and it was estimated that the angle could be set to $\pm$ 0.3° taking all sources of variation into account.

THE EXPERIMENT

THE DESIGN

It is required to measure the direct thrust, F_D , the side thrust, F_S , and the control moment, M_C produced under various controlled conditions. It is planned to vary the air reservoir pressure, p_o , the extension tube length, s and the tube inclination, θ for the two nozzle types, t described in fig.5.

Of the four controlled variables, air reservoir pressure is unique because the method of experimentation necessitates observations being made sequentially. Thus, observations of the three forces for various combinations of the other three independant variables were planned to form a factorial experiment i.e. a a factorial arrangement of tests each test containing a sequence of observations at various reservoir pressures.

The three factors and their levels are,

t - nozzle type, 2 levels.

Type 'a' designated by t_0

Type 'b' designated by t_1

s - extension length, 3 levels.

5/8" extension designated by s_0

$1\frac{1}{4}$" extension designated by s_1

$1\frac{7}{8}$" extension designated by s_2

θ - tube inclination, 4 levels.

5^o designated by θ_0

10^o designated by θ_1

15^o designated by θ_2

20^o designated by θ_3

The ranges of s and θ were chosen to include all practical values.

The entire experiment requires at least 2 x 3 x 4 = 24 tests and in addition 1/3 replication was included making a programme of (1 + 1/3) 24 = 32 tests. Further, a number of blank tests were included for control and the experiment was confounded into four randomised blocks of 8 tests each. Thus, 8 repeats are available for estimates of error and possibly 10 or more high order interactions. Of these, 3 may be confounded.

A test was planned to contain 7 observations made at 100 lbs./ins^2 intervals of p_o from 1,000 to 400 lbs./ins^2. By regression the results of the 7 factorial analyses can be correlated with p_c and the degree of freedom upon which the various estimates are made accordingly increased.

The programme is known as 'Experiment II'.

ANALYSIS OF THE DATA

The fundamental observations (as defined on page 9) of Experiment II are recorded in Table 8. The hydraulic pressures p_D, p_s and p_c correspond to the direct thrust, side thrust and control moment.

A separate analysis is necessary for each of these three statistics at each of the seven pressure levels making $3 \times 7 = 21$ analyses in all. An extension of Yates' tabular method was employed and a specimen table is shown in Table 9. The operators of the three factors are shown in Table 10.

The 1/3 replication was effected by repeating the middle level of the factor S and thereby artificially elevating it to a 4 level factor. The S operator shows one comparison, e which is a measure of error based on repeats. The inclusion of this 'dummy' level and the consequent modification of the operator requires that the mean effects be corrected before they can be applied directly to a regression equation.

The corrected mean effects are denoted by superscript letters S', etc. and are as follows:-

$$(S_0') = \frac{1}{3}\left\{3\,(S_0) + (S_2)\right\}$$

$$(S_1') = -(S_1)$$

$$(S_2') = 2/3\,(S_2)$$

The variances thus become,

$$V_{(S_0')} = V_{(S_0)} + 1/9\,V_{(S_2)}$$

$$V_{(S_1')} = V_{(S_1)}$$

$$V_{(S_2')} = 4/9\,V_{(S_2)}$$

FUNDAMENTAL DATA OF EXPERIMENT II

TREATMENT s	t	θ	TEST No.	BLOCK BIAS	CONTROL PRESS P_c lbs/ins² × 10, P_o = 400	500	600	700	800	900	1000	DIRECT THRUST PRESS P_D lbs/ins² × 10, P_o = 400 lbs/ins²	500	600	700	800	900	1000	SIDE THRUST PRESS P_S lbs/ins², P_o = 400 lbs/ins²	500	600	700	800	900	1000
0	0	0	92	ω	12	15	18	20	22	25	26	425	525	605	695	780	875	970	26	31	34	38	43	44	50
1	0	0	105	y	39	52	57	65	71	79	86	480	580	670	755	835	905	970	38	45	51	56	63	69	74
1	0	0	94	z	39	51	56	64	72	79	83	425	520	615	705	790	880	960	37	44	49	55	59	65	70
2	0	0	116	x	57	73	85	96	109	121	132	455	560	635	710	795	880	970	41	51	58	65	71	77	86
0	1	0	120	x	24	29	32	35	39	42	47	450	540	630	710	795	880	965	32	38	43	48	52	59	63
1	1	0	104	y	41	53	61	71	77	85	93	450	545	630	715	800	885	965	39	46	52	58	66	72	77
1	1	0	107	z	44	54	64	71	80	87	95	485	570	655	740	825	895	980	40	47	54	60	67	73	79
2	1	0	90	ω	64	80	91	103	118	134	144	440	530	605	700	785	870	955	42	50	58	66	73	80	88
0	0	1	98	y	32	41	50	55	62	70	76	440	540	620	710	785	875	950	43	54	61	70	79	89	97
1	0	1	118	ω	66	93	110	129	144	163	178	500	605	700	775	850	955	995	59	74	86	100	111	124	135
1	0	1	87	x	64	94	112	132	148	165	182	430	525	600	785	765	855	940	56	73	84	97	109	120	132
2	0	1	108	z	89	135	170	202	227	255	283	500	595	675	750	825	900	985	68	89	106	123	139	153	168
0	1	1	112	z	40	51	61	69	75	84	92	475	560	640	715	785	870	940	50	64	75	85	96	107	118
1	1	1	91	ω	82	104	125	143	163	182	200	440	530	605	695	770	855	935	66	81	94	107	120	134	147
1	1	1	119	x	80	101	121	140	157	174	193	465	570	650	735	820	885	980	66	82	95	108	123	137	151
2	1	1	102	y	86	106	125	143	163	184	204	445	535	610	695	770	860	935	69	84	97	111	125	139	154
0	0	2	122	x	36	53	69	81	91	102	112	470	560	635	715	795	890	970	51	72	89	103	119	133	146
1	0	2	95	z	78	120	165	196	227	252	280	445	540	620	690	770	845	920	69	100	126	144	164	181	200
1	0	2	106	y	80	123	165	194	223	250	276	515	610	685	765	840	940	975	68	102	124	147	166	185	203
2	0	2	88	ω	110	195	243	288	336	379	415	425	510	580	655	735	825	890	78	117	139	162	183	206	226
0	1	2	85	ω	49	66	82	95	107	121	133	430	510	590	670	745	820	905	63	82	97	113	126	142	155
1	1	2	111	y	110	151	199	206	233	260	284	495	575	655	745	845	870	895	91	117	138	159	180	197	219
1	1	2	100	z	115	153	184	211	239	267	291	490	570	635	705	765	845	910	95	116	138	157	176	198	217
2	1	2	115	x	74	87	101	120	142	151	163	505	590	675	750	845	865	880	83	102	119	139	158	173	190
0	0	3	109	z	51	72	93	112	131	147	164	485	555	630	700	770	840	900	67	93	115	136	155	174	192
1	0	3	84	ω	117	158	218	266	305	344	382	395	475	550	620	690	765	840	93	125	162	187	214	242	246
1	0	3	123	x	122	182	226	267	305	345	379	500	575	655	740	780	810	840	94	125	164	190	215	241	254
2	0	3	101	y	153	196	232	271	307	348	388	445	520	585	655	735	805	830	108	135	160	183	210	236	244
0	1	3	99	y	63	86	105	128	145	163	179	460	550	620	690	760	825	880	85	109	133	154	172	193	213
1	1	3	121	ω	126	161	192	224	254	285	316	470	540	615	695	790	825	850	109	138	163	188	214	238	263
1	1	3	93	x	132	167	203	238	277	301	331	415	505	585	660	725	785	830	109	139	164	189	212	240	265
2	1	3	113	z	120	147	174	204	223	248	275	445	525	605	665	715	770	840	104	128	153	176	197	216	222

TABLE 8.

EXP. No. II　　　　　　　　　　　　　　　　　　　　DATE: 9 OCT. 1951

TABLE OF ANALYSIS s, t & θ

STATISTIC :- SIDE THRUST PRESS. p_s lbs/ins²　　　　AIR PRESSURE :- 600 lbs./ins²

TREATMENT s	t	θ	TEST No.	OBS'N p_s	OPERATION (S)	(T)	[TOTAL EFFECT]	SYMBOL	DIVISOR d	MEAN EFFECT ( )	MEAN SQ. OF EFFECT		CORRECTED MEAN EFFECT	CORRECTED DIVISOR FOR MEAN
0	0	0	92	34	192	399	3,281	Σ	32	102·5	333,000	* ✓	99·8	28·8
1	0	0	105	51	207	698	0	e	16		0			
1	0	0	94	49	337	970	-243	S_1	16	-15·2	3690	* ✓	15·2	16
2	0	0	116	58	361	1,214	-207	S_2	32	-6·5	1345	* ✓	-4·4	72
0	1	0	120	43	478	0	65	T	32	2·0	132	* ✓	1·6	28·8
1	1	0	104	52	492	1	-8	eT	16		4			
1	1	0	107	54	601	2	85	S_1T	16	5·3	450	* ✓	-5·3	16
2	1	0	90	58	613	-3	-39	S_2T	32	-1·2	47·6	* ✓	-0·8	72
0	0	1	98	61	2	-39	2717	θ_1	160	17·0	46,100·	* ✓	16·4	144
1	0	1	118	86	-2	-67	-8	$e\theta_1$	80		·8			
1	0	1	87	84	2	-72	-83	$S_1\theta_1$	80	-1·0	86·4	* ✓	1·0	80
2	0	1	108	106	-1	-65	-299	$S_2\theta_1$	160	-1·9	559·	* ✓	-1·2	360
0	1	1	112	75	2	-13	-19	$T\theta_1$	160		2·3			144
1	1	1	91	94	0	-20	16	$eT\theta_1$	80		3·2			
1	1	1	119	95	-2	-82	53	$S_1T\theta_1$	80	·7	35·	xxx✓	-0·7	80
2	1	1	102	97	-1	-92	-3	$S_2T\theta_1$	160		·1			360
0	0	2	122	89	-24	15	-55	θ_2	32	-1·7	94·6	* ✓	-1·8	28·8
1	0	2	95	126	-15	24	-6	$e\theta_2$	16		2·4			
1	0	2	106	124	-45	14	-35	$S_1\theta_2$	16	2·2	77·	* ✓	-2·2	16
2	0	2	88	139	-22	12	-3	$S_2\theta_2$	32	-·1	·3			72
0	1	2	85	97	-50	-4	-11	$T\theta_2$	32		3·7			28·8
1	1	2	111	138	-22	-3	2	$eT\theta_2$	16		·3			
1	1	2	100	138	-45	-2	-17	$S_1T\theta_2$	16		18·	xxx		16
2	1	2	115	119	-20	1	65	$S_2T\theta_2$	32	2·0	132·	* ✓	1·4	72
0	0	3	109	115	-8	9	-1	θ_3	160		·0			144
1	0	3	84	162	-5	23	-6	$e\theta_3$	80		·5			
1	0	3	123	164	-3	28	-11	$S_1\theta_3$	80		1·5			80
2	0	3	101	160	-17	25	107	$S_2\theta_3$	160	·7	71·8	* ✓	0·4	360
0	1	3	99	133	-22	3	27	$T\theta_3$	160		4·6			144
1	1	3	121	163	-60	-14	2	$eT\theta_3$	80		·1			
1	1	3	93	164	-51	-38	1	$S_1T\theta_3$	80		·0			80
2	1	3	113	153	-41	10	79	$S_2T\theta_3$	160	·5	38·7	*	0·3	360
TOTAL				3,281✓	2,831✓	2934✓	5,516✓							
CHECK TOTAL				2,831	2,934	5,516								

$\Sigma \frac{[\ ]^2}{d} = 389,339$ ✓

$\Sigma p_s^2 = 389,339$

MEAN RESIDUAL :- 1·4

d/F :- 8

STD. ERROR OF SINGLE OBS'N :- ± 1·2 lbs/ins²

	F	F RES.	
1 % xxx	11·3	15·8	
0·1 % *	25·4	35·6	
✓ - 0·1 % Significant throughout			

Factor S at 3 levels, s = ⅝", 1¼" and 1⅞"
" θ at 4 levels, θ = 5°, 10°, 15° and 20°
" T at 2 levels, type a and type b

TABLE 9.

TABLE 10

OPERATORS FOR FACTORS S, T and θ.

(S)

s_0	s_1	s_1'	s_2		s/s	CHECK
+1	+1	+1	+1	S_0	4	+4
0	+1	-1	0	o	2	0
+1	0	0	-1	S_1	2	0
+1	-1	-1	+1	S_2	4	0
CHECK:-						
+3	+1	-1	+1			

(T)

t_0	t_1		s/s	CHECK
+1	+1	T_0	2	+2
-1	+1	T_1	2	0
CHECK:-				
0	+2			

(θ)

θ_0	θ_1	θ_2	θ_3		s/s	CHECK
+1	+1	+1	+1	θ_0	4	+4
-3	-1	+1	+3	θ_1	20	0
+1	-1	-1	+1	θ_2	4	0
-1	+3	-3	+1	θ_3	20	0
CHECK:-						
-2	+2	-2	+6			

<u>Note</u>;

$$\left\{ \quad V_{(x)} \quad = \quad \sigma^2/d_x \quad \right\}$$

Example.

Table 9 shows that,

$(\theta_1) \;=\; 17.0; \qquad (S_1\ \theta_1) \;=\; -1.0$ and

$(S_2\,\theta_1) \;=\; -1.9$

Hence, the corrected mean effects become,

$$(\theta_1) \;=\; 1/3\ (3 \times 17.0 - 1.9) \;=\; 16.4$$

$$(S_1\,\theta_1) \;=\; 1.0$$

$$(S_2\,\theta_1) \;=\; 2/3\ (-1.9) \;=\; -1.2$$

and the variances,

$V_{(\theta_1)} \;=\; \sigma^2/9 \times 16$; Corrected divisor for mean = 144

$V_{(S_1\,\theta_1)} \;=\; \sigma^2/80$; Corrected divisor for mean = 80

$V_{(S_2\,\theta_1)} \;=\; \sigma^2/360$; Corrected divisor for mean = 360

These corrections are listed in Table 9.

Henceforth, unless otherwise stated, 'mean effect' will refer to the corrected effect and the superscript will be dropped.

THE EFFECT OF CONFOUNDING

If blocks of tests are biased by constant amounts w, x, y and z (as indicated in Table 8), twenty three effects will be free of bias and the eight confounded effects will be biased as follows:

TOTAL EFFECT	AMOUNT OF BIAS
$[S_2\,\theta_1]$	8 (-w-x+y+z)
$[eT\,\theta_1]$	8 (-w+x-y+z)
$[S_1T\,\theta_1]$	- 8 (-w+x-y+z)
$[eT\,\theta_2]$	4 (-w+x+y-z)
$[ST\,\epsilon_2]$	4 (-w+x+y-z)
$[S_2\,\epsilon_3]$	16 (-w-x+y+z)
$[eT\,\epsilon_3]$	- 4 (-w+x-y+z)
$[S_1T\,\epsilon_3]$	4 (-w+x-y+z)

The eight effects which contain the term 'c' are 'pure' expressions of error based upon repeats and since three of these are confounded, the presence of block bias may be detected by comparisons amongst these groups.

These tests for confounding showed control moment and side thrust to be free of block bias but direct thrust was found to be seriously confounded. The reason for this was traced to inadequate lubrication of the upper part of the reaction balance and a consequent deterioration which only showed itself after the initial tests of Experiment I. This was an unexpected and disappointing result but a modification is expected to overcome this defect in any future work.

Since tests were confounded in four blocks, there are only three independant confounded groups (excepting the mean). Hence, five of the confounded effects may be 'de-confounded' if the remaining three effects used as 'keys' may be assumed to be expressions of error and confounding only.

Inspection of the biased effects set out on page 17 shows what alternatives are available. $cT\epsilon_1$, $cT\epsilon_2$ and $S_2\epsilon_3$ were chosen as 'keys' and lead to the following expressions for the 'de-confounded' effects and their variances,

$(S_2\epsilon_1) = (S_2\epsilon_1)^- - \frac{1}{2}(S_2\epsilon_3)^-$ Variance $= \sigma^2/128$

$(S_1T\epsilon_1) = (S_1T\epsilon_1)^- + (cT\epsilon_1)^-$ Variance $= \sigma^2/40$

$(S_1T\epsilon_2) = (S_1T\epsilon_2)^- - (cT\epsilon_2)^-$ Variance $= \sigma^2/8$

$(cT\epsilon_3) = (cT\epsilon_3)^- + \frac{1}{2}(cT\epsilon_1)^-$ Variance $= \sigma^2/64$

$(S_1T\epsilon_3) = (S_1T\epsilon_3)^- - \frac{1}{2}(cT\epsilon_1)^-$ Variance $= \sigma^2/64$

(The superscript $^-$ refers to a confounded effect).

By this process, only three unimportant effects are irretrievably lost. The logic of these steps calls for further explanations but these would be outside the scope of this treatment.

Henceforth, where applicable, only effects which are free of confounding, or have been 'de-confounded', will be compared.

THE ESTIMATION OF ERROR

A homogeneity test was first made by comparing the high order interactions with the effects of repeats. If these formed a homogeneous set of variances, the high order interactions were included in the estimate of error; otherwise, they were rejected and the error variance based only on repeats.

A second test was made on the variances accepted from the first test to ensure that these were uniform at all air pressure levels.

As a result of these tests, it was found that nine high order interactions could be included in the estimate of direct thrust variance giving 15 degrees of freedom at each pressure level. The error variance of control moment and side thrust were restricted to estimates based on repeats only and this reduced the degrees of freedom to eight at each air pressure level.

These variances were used to test the significance of the various effects at each air pressure level. In order to reduce the complexity of the results, the 0.1% probability level was adopted in the case of the control moment and side thrust. This more critical level was also considered more suitable for these cases where the degrees of freedom available for estimates of error were rather limited. The 1% probability level was considered suitable for the direct thrust.

The significant mean effects found at each pressure level were correlated with air pressure by means of regression equations, thus treating air pressure as a completely independant variable. The residuals from these regressions were first shown to be not significantly different from the variances obtained as described above. These residuals were then pooled with this variance to obtain for each statistic an overall estimate of error based on a large number of degrees of freedom.

The correlated mean effects are tabulated in table 11. The final pooled variances are summarised in table 13.

TABLE 11

SUMMARY OF CORRELATED RESULTS

SIDE THRUST GAUGE PRESSURE

(Σ)	=	$-12.1 + .213\, p_o - .0000436 p_o^2$	Std. DEVIATION = $\pm .44$ (124d/F)
(S_1)	=	$4.1 + .0185 p_o$	$\pm .60$
(S_2)	=	$.16 - .007\, p_o$	$\pm .28$
(θ_1)	=	$-8.71 + .0517\, p_o - .0000165\, p_o^2$	$\pm .20$
$(S_1 \theta_1)$	=	1.0	$\pm .27$
$(S_2 \theta_1)$	=	$.52 - .00268\, p_o$	$\pm .13$
(θ_2)	=	$2.37 - .00715\, p_o$	$\pm .44$
$(S_1 \epsilon_2)$	=	$2.75 - .00821\, p_o$	$\pm .60$
$(S_2 \epsilon_3)$	=	$.09 + .0005\, p_o$	$\pm .13$
(T)	=	1.9	$\pm .44$
$(S_1 T)$	=	$.57 - .0091\, p_o$	$\pm .60$
$(S_2 T)$	=	-1.1	$\pm .28$
$(S_1 T \theta_1)$	=	$-.7$	$\pm .27$
$(S_2 T \theta_2)$	=	$.6$	$\pm .28$

DIRECT THRUST GAUGE PRESSURE x 10

(Σ)	=	$82.3 + 1.007\, p_o - .000165\, p_o^2$	STD. DEVIATION = ± 3.78 (119 d/F)
(θ_1)	=	$14.36 - .0306\, p_o$	± 1.69
(θ_2)	=	$-3.0 - .01\, p_o$	± 3.78

CONTROL MOMENT GAUGE PRESSURE x 10

(Σ)	=	$-35.3 + .299\, p_o - .0000707\, p_o^2$	STD. DEVIATION = ± 1.03 (140 d/F)
(S_1)	=	$-.1 + .074\, p_o$	± 1.37
(S_2)	=	$1.21 - .0188\, p_o$	$\pm .65$
(θ_1)	=	$-3.57 + .0364\, p_o$	$\pm .46$
$(S_1 \epsilon_1)$	=	$1.5 + .0029\, p_o$	$\pm .62$
$(S_2 \theta_1)$	=	$1.2 - .0072\, p_o$	$\pm .29$
(ϵ_2)	=	$3.83 - .0124\, p_o$	± 1.03
$(S_1 \epsilon_2)$	=	$8.0 - .0173\, p_o$	± 1.37
(T)	=	$11.1 - .0282\, p_o$	± 1.03

TABLE 11 CONTINUED

CONTROL MOMENT GAUGE PRESSURE x 10

(S_1T)	=	$6.3 - .0387\ p_o$	$\pm$ 1.37
(S_2T)	=	$-1.71 - .00479\ p_o$	$\pm$ .65
$(T\Theta_1)$	=	$2.67 - .00918\ p_o$	$\pm$ .46
$(S_1T\Theta_1)$	=	$1.06 - .0078\ p_o$	$\pm$ .62
$(S_1T\Theta_2)$	=	$-5.82 + .0213\ p_o$	$\pm$ 1.37
$(S_2T\Theta_2)$	=	$-1.25 + .00946\ p_o$	$\pm$ .65
$(S_1T\Theta_3)$	=	$-1.3 + .0066\ p_o$	$\pm$ .62
$(S_2T\Theta_3)$	=	$.11 + .00189\ p_o$	$\pm$ .29

THE RESULTS

PRESENTATION OF RESULTS

It should be noted that the figures so far quoted refer to the hydraulic pressures set up by the control moment, side thrust and direct thrust. They may easily be converted into the desired units by use of the calibration results given on page 10.

The mean effects which have been shown to be significant may readily be combined to form a polynomial expressing the statistic in terms of s, θ, t and p_o. This process is described in Appendix II.

So many interactions have proved significant in the case of side thrust and control moment that the resulting expressions are extremely cumbersome and hence of little practical utility.

However, the full regression equation is given for direct thrust (see p.26).

Because of the unwieldy nature of the regression equations, the 'most probable' values of the results are tabulated (table 12) and these also presented in graphical form (Figs. 20-26).

It must be kept in mind that the complexity of the results and the consequent difficulty of presentation is a reflection of the complexity of the process investigated. No simplification is possible without equivalent loss of accuracy and it is felt that this would only conceal valid information whilst the accuracy lost would be greater than that lost by reading values from the graphs.

Precision of the Results.

On page 18 it was shown how the variance was derived for each statistic. These are the variances of single observations and expressed in the original units of hydraulic pressure. The variances of the 'most probable' results are simply derived from these and are listed in table 13. These are an index of the precision of the figures quoted and thus a measure of the reproducibility of results with the apparatus used.

For design purposes the fiducial limits may be required. These indicate the likely range of an estimated result taking all experimental trends into account. This may be calculated from the regression equation and is illustrated by an example in Appendix II.

It is seen from table 13 that for control moment and side thrust, the experimental variances are not significantly different from the calibration errors given on page 10. This fact gives an indication of the high quality of the experimentation.

TABLE 12

SUMMARY OF 'MOST PROBABLE' RESULTS

DEFLECTION			SIDE THRUST F_s lbs.				CONTROL MOMENT M_c lbs. ft.			
TUBE LENGTH (ins.)	ANGLE	TYPE								
		p_o =	400,	600,	800,	1000,	400,	600,	800,	1000, lbs/ins^2
5/8	5°	a	8.0	11.8	16.0	20.4	-.23	0	.12	.13
10/8	5°	a	13.6	19.0	25.7	32.2	.20	.64	.95	1.16
15/8	5°	a	17.6	25.3	33.5	42.0	.73	1.36	1.87	2.28
5/8	10°	a	16.6	27.3	37.0	45.4	.10	.51	.80	.97
10/8	10°	a	22.7	38.1	52.5	65.7	.81	1.72	2.53	3.23
15/8	10°	a	30.6	48.9	66.1	82.1	1.37	2.75	3.99	5.16
5/8	15°	a	23.0	40.3	55.2	67.4	.27	.78	1.18	1.46
10/8	15°	a	35.6	60.2	82.2	101.2	1.21	2.71	3.74	4.81
15/8	15°	a	40.6	65.5	87.8	107.5	2.25	4.21	6.06	7.81
5/8	20°	a	30.3	55.1	75.7	92.6	.45	1.40	2.23	2.94
10/8	20°	a	46.1	77.3	104.2	127.7	2.13	4.03	5.85	7.51
15/8	20°	a	50.6	79.2	104.0	125.0	2.47	4.20	5.85	7.34
5/8	5°	b	10.5	16.2	22.3	28.6	-.14	.11	.27	.30
10/8	5°	b	16.7	22.6	28.8	35.2	.48	.86	1.12	1.28
15/8	5°	b	18.0	23.8	30.2	36.7	.71	1.34	1.88	2.28
5/8	10°	b	19.3	31.9	43.5	53.8	.24	.66	.99	1.20
10/8	10°	b	28.2	43.7	57.9	71.1	1.12	2.04	2.73	3.51
15/8	10°	b	28.3	44.7	60.0	74.2	1.03	1.92	2.70	3.36
5/8	15°	b	27.2	46.3	63.2	77.2	.60	1.35	1.99	2.51
10/8	15°	b	41.1	65.7	87.8	107.2	1.87	3.20	4.42	5.52
15/8	15°	b	36.8	59.8	80.2	98.2	1.13	1.93	2.63	3.20
5/8	20°	b	37.3	63.9	86.4	105.2	.78	1.60	2.30	2.89
10/8	20°	b	49.1	80.3	107.2	130.7	2.02	3.38	4.62	5.75
15/8	20°	b	46.5	73.3	96.2	115.2	1.87	2.94	3.91	4.75

TABLE 12 CONTINUED

SUMMARY OF 'MOST PROBABLE' RESULTS (CONTINUED)

DEFLECTION ANGLE, θ	DIRECT THRUST F_D lbs.			
	p_c = 400	600	800	1000 lbs./ins.^{2}
5°	108.0	166.0	219.5	270.0
10°	113.5	169.0	220.5	268.0
15°	114.5	166.5	214.0	257.5
20°	111.5	158.3	201.0	239.0

TABLE 13

SUMMARISED ERROR VARIANCES

STATISTIC	OBSERVED VARIANCE x (of a single observation) Original Units	Force ж Units	Degrees of Freedom	VARIANCE OF "Most Prob." RESULTS	STANDARD DEVIATION OF "Most Prob." RESULTS
SIDE THRUST	5.68	1.57	124	1.18	± 1.1 lbs.
DIRECT THRUST	4.12	41.0	119	5.12	± 2.26 lbs.
CONTROL MOMENT	.303	.0124	140	.0093	± .096 lbs.ft

x This is the final pooled variance obtained as discussed on p. 18.

ж Since $F_s = 0.525\, p_s - 5.77, \quad V_{(F_s)} = (.525)^2 \; V_{(p_s)}$

$$= (.525)^2 \times 5.68 = 1.57$$

and similarly for the other statistics.

DIRECT THRUST

The very high error variance of the direct thrust observations at least makes for a simple regression equation:

$$F_D = -31.0 + .35p_o - .000052p_o^2 + 2.75\theta - .0378\theta^2$$
$$- .000705\, p_o\theta - .000126\, p_o\theta^2 \quad \text{lbs.}$$

This expression is plotted in Fig.24. When $\theta = 0$, the thrust reduces to,

$$F_{D\,(\theta = 0)} = -31.0 + .35\, p_o - .000052\, p_o^2 \quad \text{lbs.}$$

Neglecting the second order term gives the following linear relationship,

$$F_D \doteq -10.8 + .277\, p_o \text{ lbs.}$$

If this expression is corrected for the thrust generated by acceleration in the stand-pipe at the foot of the reaction balance, it becomes,

$$F'_I \doteq -10.8 + .2859\, p_o \quad \text{lbs.}$$

This compares very favourably with the mean theoretical undeflected thrust for the two nozzle types.

$$F_D'' = .2865\, p_o \text{ lbs. at } T_o = 300^{\circ}K$$

This gives a thrust efficiency at 0° deflection of 94.2% at $p_o = 700$ lbs./ins.^{2}

The regression equation approximates closely to the known boundary conditions i.e. when $p_o = 0$ and $\theta = 0$.

If the quadratic term in p_o is admitted to have meaning (it has been proved to be statistically significant), an optimum air pressure may be deduced as follows:

$$\frac{\partial F_D}{\partial p_o} = .35 - .000104\, p_o - .000705\,\theta - .000126\,\theta^2 = 0$$

whence, $\quad p_o = 3,360 - 6.8\,\theta - 1.2\,\theta^2$ for max. F_D

Thus, with no deflection ($\theta = 0^{\circ}$) the optimum reservoir pressure is 3,360 lbs./ins.^{2} and this value falls <u>slightly</u> with increasing angle of deflection to 3,172 lbs./ins.^{2} at $\theta = 10^{\circ}$ and 2,744 lbs./ins.^{2} at $\theta = 20^{\circ}$.

However, it must be kept in mind that these figures lie well outside the experimental range and hence should be accepted with caution.

The variance of F_D is much larger than those of the other statistics, as can be seen in table 13. This is not unexpected for reasons given on page 18. It must be emphasised though, that although individual values for direct thrust have a low precision, the regression equation presents the true trends i.e. it gives the true quantitative dependance of thrust on deflecting angle and reservoir pressure. Better precision may have resulted in an additional dependance on length of tube.

SIDE THRUST

The derived polynomial expressing F_s as a function of p_o , S, θ and t is too complicated to permit of a rigorous analytical consideration of the possible maxima existing. However, some simplification is possible.

Inspection of the general expression or of the graphs in Figs.20–26 shows that there is no useful maximum θ. In practice the most likely problem will be to find the combination of p_o , s and t which yield a max. F_s for a given θ.

Since t is not a continuous variable, separate analyses must be applied to each type. F_s is linear in p_s and so there is no need to transform the equation from its original units.

For type 'a' with θ = 20°,

$$\frac{\partial p_s}{\partial s} = -32.72 + .3102\, p_o + 35.48\, s - .2233\, p_o s.$$

Inspection of this equation shows that sensible values of p_o and s are non-concomitant if the derivative is to equal zero. Hence, there is no useful solution to the problem of finding the optimum p_o : s combination. However, if p_o is chosen from other considerations, the optimum tube length is readily determined.

These conditions are found to hold for both types at all angles. Optimum values of s for various choices of the other variables are tabulated below.

These values all lie within the range studied and hence their reliability can be accurately assessed.

FOR MAXIMUM F_s

	TYPE 'a'		TYPE 'b'	
θ =	0°	20°	0°	20°
p_o = 400 lbs./ins.^{2}	s = 1.28"	1.70"	1.31"	1.36"
p_o = 800 lbs./ins.^{2}	1.24"	1.50"	1.18"	1.39"
p_o = 1000 lbs./ins.^{2}	1.23"	1.48"	1.15"	1.36"
p_o = optimum for max. F_D (see p. 26)	1.17"	1.42"	1.02"	1.29"

CONTROL MOMENT

This statistic follows a broadly similar pattern to that of side thrust but the effect of type and its interactions are more predominant (see table 11). A similar treatment to that of the preceding section may be applied to the regression equation for M_c but inspection shows that there is no minimum and there appears to be no practical reason for determining accurately conditions leading to a maximum. Approximate values may be readily obtained from the graphs - Figs.20 - 26.

Considering the practical utility of the information, in any application involving a permanent predetermined jet deflection, an approximate value for the maximum force likly to occur is all that would be required. Since M_c is not linear in θ and furthermore has maxima for many conditions, any automatic control of jet deflection by a system which responds to direction signals by applying a given control moment would be useless. It would be necessary to employ a system which responds by setting the tube to a required angle (the system discussed by Friedman, ref.3). Side thrust is sensibly linear in θ over wide ranges which simplifies the problem and it is only necessary to know the power requirement to overcome the maximum control moment.

THE IMPRACTICABILITY OF ASSESSING EFFICIENCY

It would be useful to estimate the thrust efficiency of the nozzle under various conditions of deflection but the high error variance of individual direct thrust measurements makes useful estimates of this quantity impossible.

Another useful efficiency estimate would be a comparison of the vector sum of side and direct thrust with the direct thrust of an undeflected jet. (See fig.13.a.).

However, the error variance of this ratio, dependent as it is on the variance of direct thrust, is so great that a factorial analysis shows all factors to be non-significant. For example, when p_o = 1,000 lbs./ins.^{2}, the variance of the resultant thrust is 37.6 (see Appendix II) and the overall mean sum of squares (of deviations from the mean) is only 34.9 for this statistic.

This is an unfortunate consequence of the feature of the direction reaction balance mentioned on page 18 and it cannot be overcome by any analytical technique.

THE LINE OF THRUST OF THE DEFLECTED JET

The vector combination of direct and concomitant side thrust enables the line of resultant thrust action to be estimated. Let the angle this makes to the normal axis be ϕ (See Fig.13.a.).

An analysis of the variance of ϕ is made in Appendix II and there it is shown that the coefficient of variation is very small.

	Coefficient of Variation,		
	F_s	F_D	ϕ
p_o = 1,000 lbs./sq.in.	1.59%	2.48%	.0485%
= 400 " "	4.25%	5.7%	.119%

Thus, in spite of the high variance of F_D, ϕ may be determined quite accurately (within the experimental range).

This parameter ϕ has been evaluated from the "most probable" values of F_s and F_D at p_o = 1,000, 800, 600 and 400 lbs./sq.in. at all the experimental levels. From these sets of results a factorial analysis has been carried out. This leads to the regression equation,

$$\phi = 2.2 - .0035\,p_o - 5.1s + 3.83s^2 - .04\,\theta$$
$$+ .00058\,p_o\,\theta - .6144s^2\theta + 1.54s\,\theta$$
$$+ 3.2\,t - 2.6\,st$$

Note that it is linear in all terms but S (t takes the value 0 for type a and +1 for type b). It is set out in graphical form - Figs. 25 and 26.

Inspection of this equation shows that there is no optimum p_o or .
However, $\dfrac{\partial\phi}{\partial S} = 0$

when,

$$S = \frac{5.1 - 1.54\,\theta}{7.66 - 1.23\,\theta} \quad \text{for type a}$$

$$= \frac{7.7 - 1.54\,\theta}{7.66 - 1.23\,\theta} \quad \text{for type b}$$

Note:- $\dfrac{\partial^2\phi}{\partial S^2}$ is negative only for $\theta > 6.25^\circ$

and thus maximum ϕ exists only when $\theta > 6.25^\circ$

At $\theta = 10^\circ$, $S_{OPT.}$ = 2.25" for type a
= 1.66" " " b

It may be noted that these values are greater than those required to produce maximum side thrust (see page 27).

THE MECHANISM OF DEFLECTION

The mechanism of deflection by inclined tubular extensions appears to be a fairly simple case of momentum change. A series of shadowgraphs have been studied and these, in conjunction with the foregoing data, have enabled a qualitative description of the mechanism of deflection to be formulated.

No new shock wave patterns of any intensity are set up by the deflector. The extension appears to act as a "momentum guide" and has a "smoothing" effect upon the jet. Expansion in the conical divergent portion of the nozzle increases the normal momentum but also causes a small amount of tangential momentum which causes the jet to continue expanding even if it has reached atmospheric pressure at the exit plane. Thus, over-expansion occurs followed by some recompression (by the atmosphere) and the characteristic shock "X" pattern is set up. This is a periodic phenomenon and though highly damped by the turbulent boundary, a second shock "X" may often be observed (Fig.14, Plate 17).

The tubular extension (when not inclined) prevents over expansion by tangential momentum and a more parallel jet results. The momentum change effected by the tube sets up a similar shock "X" as before but now plainly originating from the region of application of the necessary force. If the jet is not correctly expanded, a second shock "X" will originate from the exit plane of the tube. (Fig.15 and Plate 18).

The growth of a boundary layer in the tube may assist recompression. It is notable that the "momentum guidance" imparted by a fairly long tube will serve to keep an under-expanded jet parallel for some distance outside the nozzle before it expands completely.

If the tube is inclined, the upper part of the jet will now be subjected to much more severe "momentum guidance" and there will be an even more intense zone of re-compression set up by the upper part of the deflector. The lower part of the jet, because of its normal momentum, will detach from the tube wall and add to the re-compression.

If the tube is fairly long, the second expansion will be restricted in one direction by the upper surface of the deflector and asymmetric expansion will produce a resultant momentum inclined at a greater angle than the tube walls. (Fig.16 and Plate 19).

Beyond an optimum tube length, a second recompression may occur. Expansion after this may result in a more symmetrical jet and hence less difference between the angles θ and $\emptyset$. Thus, this optimum tube length corresponds to that which will produce max. $\emptyset$ for a given θ and, if efficiency is 100%, to that which will produce max. F_s for the same θ. If the tube is too short to complete the first recompression, the jet will not be turned through the full angle θ.

The conical enlargement at the exit of nozzle type 'a' will increase the tangential component of momentum and thus the "duty" that the tube must perform. This will tend to increase the control moment and, in general, type 'a' was found to have the higher control moment and smaller side thrust (see graph Fig.23). Type 'b', as expected, was found to be more generally efficient.

The formation of the jet in the nozzle does not appear to be hindered in any way by the tubular extension and so this method of jet deflection may be classified as "diversion" (see ref.1).

CONCLUSIONS AND DISCUSSION OF RESULTS

The "most probable" results, as obtained on pages 23 - 24 , are presented graphically (together with error ranges) in figs. 20 - 26. They apply to an air jet discharged from a $\frac{1}{2}$" model nozzle at a Mach No. of about 3.0.

Side thrust is found to increase fairly regularly with increasing inclination of the extension tube and with reservoir pressure, but the details of this variation are rather complex. The two types yield different results in detail though they behave in a broadly similar manner. At higher deflection angles, optimum tube lengths lead to maximum side thrust. The maximum side thrust produced was about 50% of the direct thrust and the smallest value recorded was near 10%.

The direct thrust appeared to be not seriously reduced when the jet was deflected. The high error variance of this statistic makes efficiency assessments impossible but dependence upon tube inclination and reservoir pressure has been established.

Control moment varies fairly regularly with angle of inclination and reservoir pressure though even more complex in detail than side thrust. Variation with tube length and type is both complex and irregular. The maximum moment recorded was less than 8 lbs. ft. Most of the results were found to be less than 4 lbs. ft. In rough figures, the work needed to turn the mid-sized tube from 0 - 20° at 800 lbs./sq.in. reservoir pressure was 1 ft. lb.

The differences due to type were smaller than those caused by other factors. The two types required particularly different control moment especially at the upper limits of the factors. In general, type "b" produced the greater side thrust and required the smaller control moment.

The inclination of the line of thrust was found to vary linearly with tube inclination and air pressure, though the latter variation was very small. Type had little effect on this but tube length occurred as a quadratic effect. These are convenient results from the point of view of application. Except when short tube lengths were used, the resultant line of thrust was turned through a greater angle than the tube inclination. Graphs Figs. 25 - 26 set out these results.

Analysis of the observations results in optimum tube lengths which may be chosen depending upon whether it is required to produce maximum side thrust for a given inclination or maximum inclination of the line of thrust. These optimum lengths are of the order of three times the nozzle throat diameter.

The optimum tube lengths produce a side thrust of about 8 times the average value produced by a vane set at the same angle. The corresponding control moment is about 3 times that required for a vane. (see ref. 1).

The shorter tube lengths (which may in practice be more desirable) produce about 5 times the side thrust of a vane at the same angular setting and require a similar control moment.

Provided that other requirements are met (weight, corrosion resistance etc.), the tubular extension method provides a reliable means of deflecting supersonic jets which, on the laboratory scale, appears to be much more efficient than any other method so far reported.

REFERENCES

1. P. Rowe; "Supersonic Jet Deflection - Part I. Methods of Jet Deflection and a Review of Previous Work". M.O.S. Extra Mural Research Report PDGW/EMR/4 (Imperial College Report JRL 24) September 1952.

2. M.O. Coulter; "Losses in Conical and Annular Supersonic Nozzles with Special Reference to the Loss due to Separation and Shock at a Badly Designed Throat". M.O.S. Extra Mural Research Report PDGW/EMR/52/1 (Imperial College Report JRL 16) May 1952.

3. H. Friedman; "Summary Report on A.4. Control and Stability". Report No. F-Su-2152-ND by H.Q. Air Material Command, Dayton, June 1947.

4. Fairey Aviation Co: R.A.E. Supersonics Sections and J.R.L. etc. Unpublished papers and notes of meetings.

5. P. Eisenklam "Statistical Methods in Engineering Experimentation" Research 6. (~~March~~ 1953) (In Press)

APPENDIX I

CALCULATION OF THE FLOW CHARACTERISTICS OF NOZZLE 'b'

If it is assumed that air behaves as a perfect gas, γ = 1.4 and that it expands adiabatically, the conditions of flow within the nozzle can be estimated.

For an ideal nozzle of $\frac{1}{2}$" diam. throat discharging air to atmosphere, from a reservoir at T_0 = 300°K (an average value), mass flow rate,

$$m = .00449\ p_0 \text{ lbs./sec.}$$

Throat Velocity (sonic), U_t = 1,040 ft./sec.

For nozzle type 'b',

$$M_e = 2.81$$

$$u_e = 2{,}000 \text{ ft./sec.}$$

Thrust,

$$F_D = \frac{m\ U_e}{g}$$

$$= .279\ p_0 .$$

$$\text{H.P.} = .564\ p_0 .$$

Design pressure = 400 lbs./ins.^{2}

The data for nozzle type 'a' are similar but it is necessary to make assumptions about the point of detachment before they can be calculated.

APPENDIX II

CALCULATION OF FIDUCIAL LIMITS

Suppose nozzle type 'a' has been chosen to operate at a reservoir pressure, p_o of 1,000 lbs./ins.^{2} and it is required to determine the maximum side thrust which may be obtained at a 20° deflection angle. On page 27 it is shown that the optimum extension length for these conditions is 1.48". From table 11, the mean effects for p_o = 1,000 lbs./ins.^{2} can be evaluated and the regression equation expressing p_s in terms of s, θ, and t may then be built up.

Thus,

$$p_s = (\Sigma) + (S_1)\,\xi_1'(s) + (S_2)\,\xi_2'(s) + (\theta_1)\,\xi_1'(\theta)$$

$$+ (S_1\theta_1)\,\xi_1'(\theta)\,\xi_1'(s) + \text{etc.}$$

where the polynomials in s, θ and t are given by,

$$\xi_1'(s) = 1/5\,(8s-10) \qquad \xi_2'(s) = 1/25\,(192s^2 - 480s + 250)$$

$$\xi_1'(\theta) = 2/5\,(\theta - 12.5) \qquad \xi_2'(\theta) = \left(\frac{\theta^2}{25} - \theta + 5\right)$$

$$\xi_3'(\theta) = 10/3\left(\frac{\theta^3}{125} - \frac{3\theta^2}{10} + 3.34\theta - 10.5\right)$$

$$\xi_1'(t) = 2\left(t - \tfrac{1}{2}\right)$$

(see Fisher and Yates' 'Statistical Tables')

In the example,

$$p_s = 157.3 + 22.6 \times .36 + 6.84 \times 1.6 + 26.5 \times 3.0$$

$$+ 1.0 \times .36 \times 3.0 + \text{etc.}$$

$$= 260.76 \text{ lbs./ins.}^2$$

From the calibration equation of page 10,

$$F_s = .525 \times 260.76 - 5.77 = \underline{\underline{131.2 \text{ lbs. force}}}$$

The fiducial limits are given by,

$$V_{(p_s)} = V_{(\Sigma)} + .36^2 V_{(S_1)} + 1.6^2 V_{(S_2)} + \text{etc.}$$

$$= \left(\frac{1}{d_\Sigma} + \frac{.36^2}{d_{s1}} + \frac{1.6^2}{d_{s2}} + \text{etc.}\right) \times S^2_{p_s}$$

$$= .3980 \times 5.68 = 2.26$$

therefore, $V_{(F_s)} = .525^2 \times 2.26 = .622 = .788^2.$

Thus, the standard error of $F_s = \pm .788 \simeq \pm .8$

and the required result is $\underline{\underline{F_s = 131.2 \pm .8 \text{ (124 d/f) lbs. force.}}}$

APPENDIX III

THE VARIANCE OF THE RESULTANT THRUST, F_R .

$F_R = \sqrt{F_D^2 + F_S^2}$ and if F_D and F_S are independent quantities,

$$V_{(F_R)} = \frac{F_D^2}{F_R^2} V_{(F_D)} + \frac{F_S^2}{F_R^2} V_{(F_s)}$$

Now, from table 13,

$$V_{(F_D)} = 41.0 \text{ and } V_{(F_s)} = 1.57$$

Taking mean values for F_s and F_D at various pressure levels,

@ p_o = 1,000 lbs./sq.in., $V_{(F_R)}$ = 37.6

$\therefore F_R = 269.5 \pm 6.13$, v = 2.28%.

@ p_o = 400 lbs./sq.in. $V_{(F_R)}$ = 38.5

$\therefore F_R = 115.4 \pm 6.2$, v = 5.4%

THE VARIANCE OF THE RESULTANT DEFLECTION, ϕ

$\phi = \tan^{-1} F_s/F_D$ and if F_D and F_s are independent quantities,

$$V_{(\phi)} = \frac{F_D^2}{F_R^4} V_{(F_s)} + \frac{F_s^2}{F_R^4} V_{(F_D)}$$

Taking mean values and the known variances as before.

@ p_o = 1,000 lbs./sq.in., $V_{(\phi)}$ = .000068

$\therefore \phi = 17.0^\circ \pm .00824$, v = .0485%

@ p_o = 400 lbs./sq.in., $V_{(\phi)}$ = .000309

$\therefore \phi = 14.5^\circ \pm .0176$, v = 0.119%.

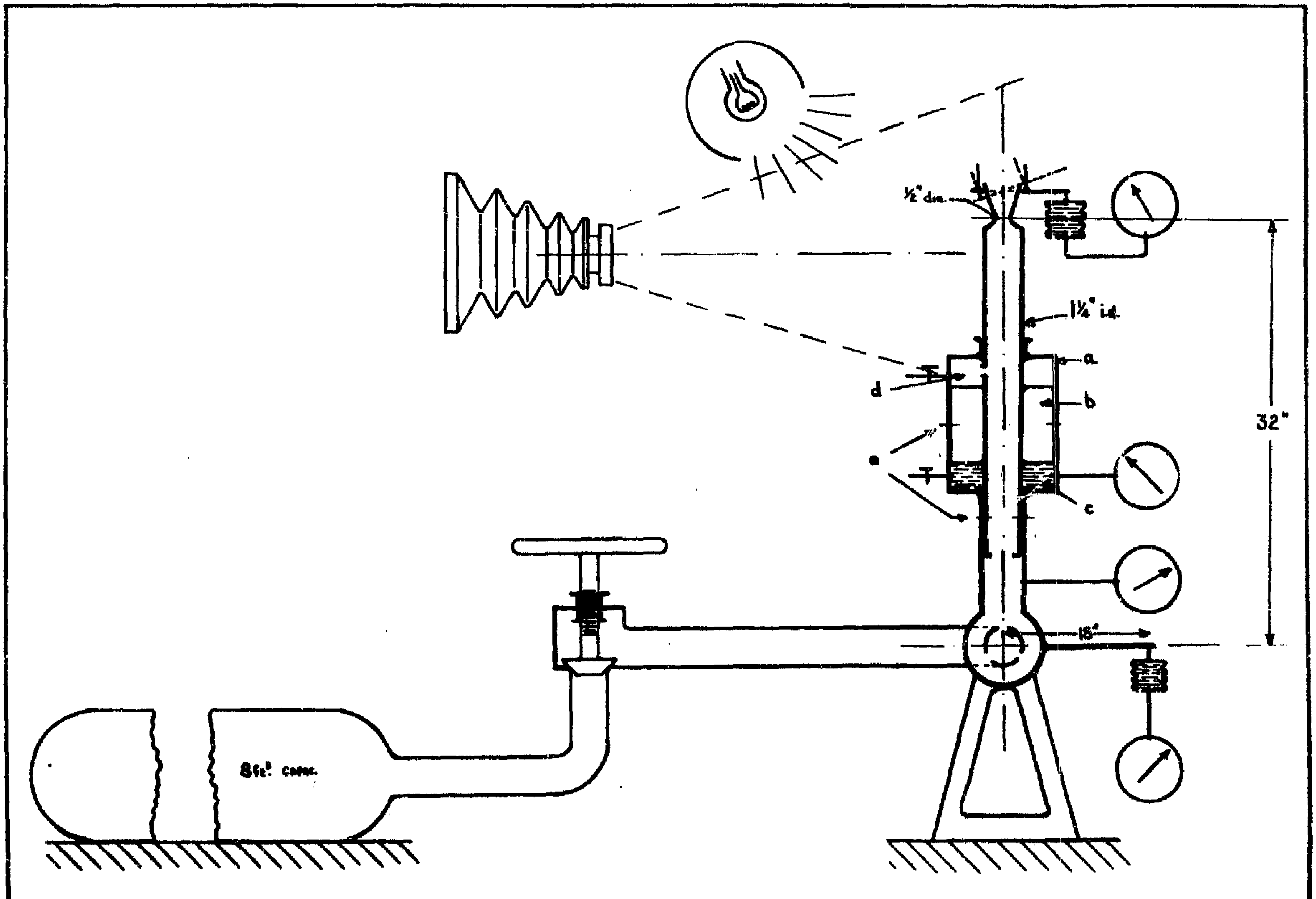
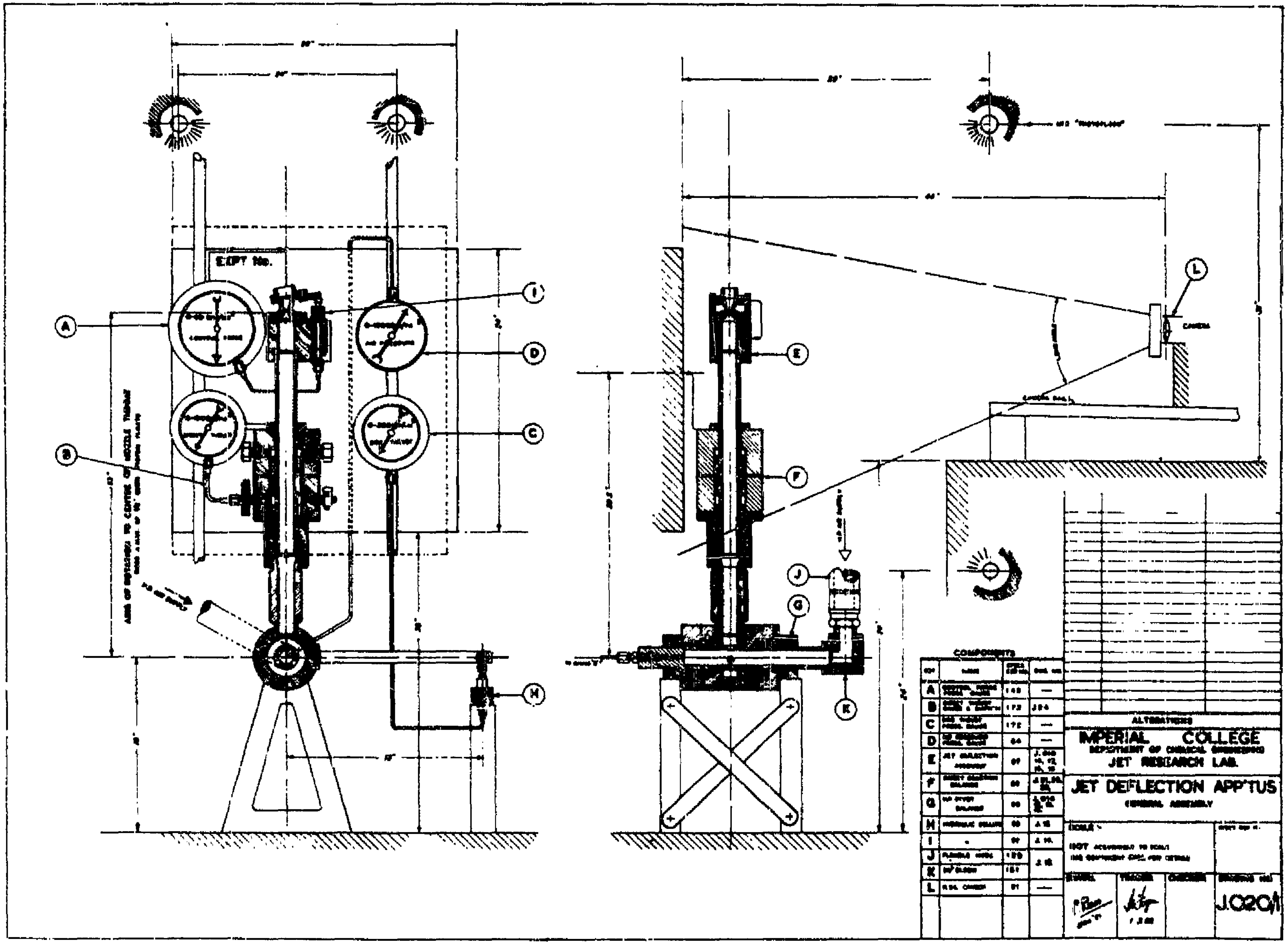
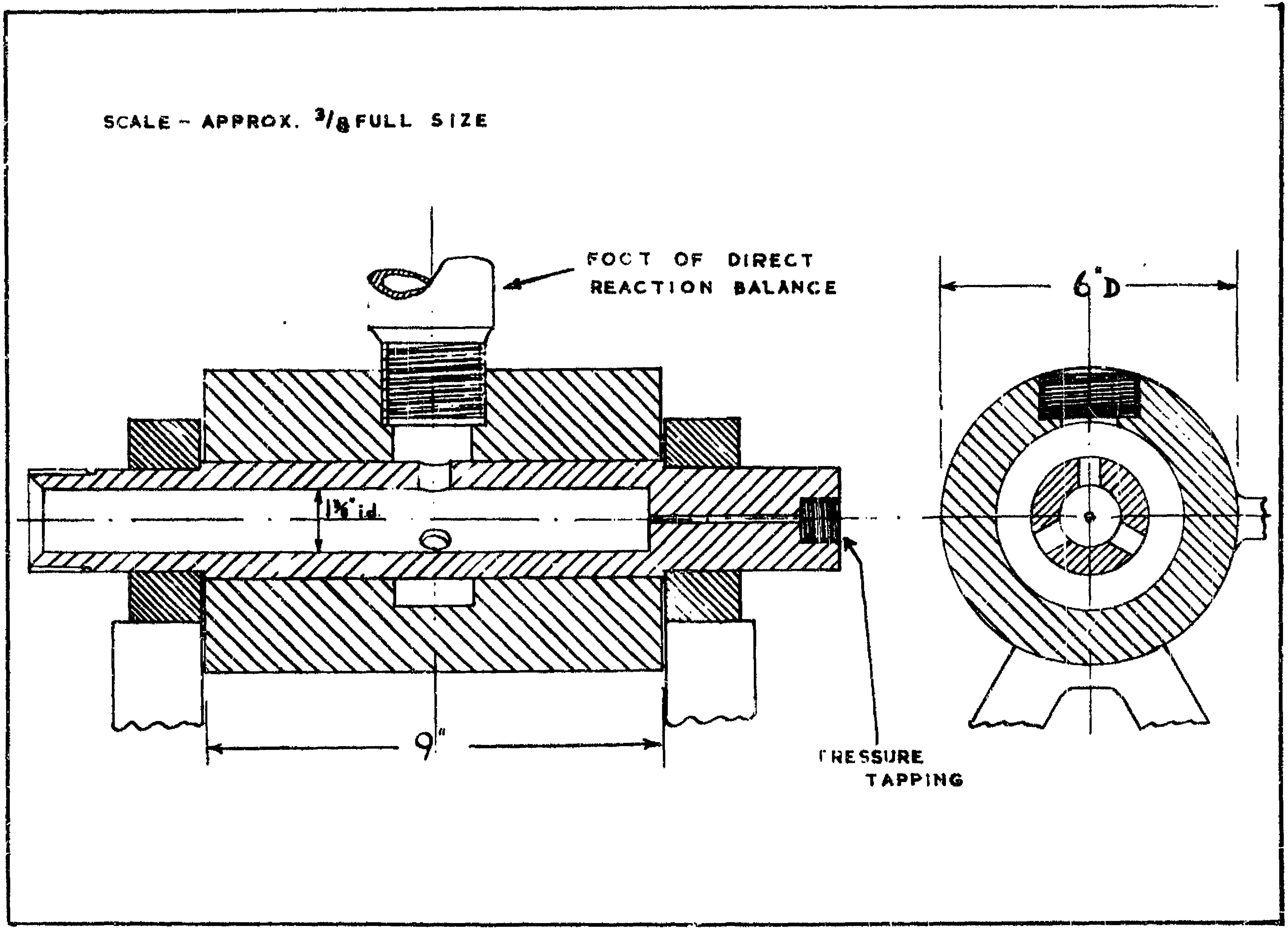


FIG. 1.



EXPT No.
A
B
C
D
E
F
G
H
I
J
K
L
CAMERA
COMPONENTS
IMPERIAL COLLEGE
JET RESEARCH LAB.
JET DEFLECTION APP'TUS
J.C2O/1

FIG. 2.



SCALE - APPROX. 3/8 FULL SIZE
FOOT OF DIRECT REACTION BALANCE
6" D
1⅜" i.d.
9"
PRESSURE TAPPING

FIG. 3.

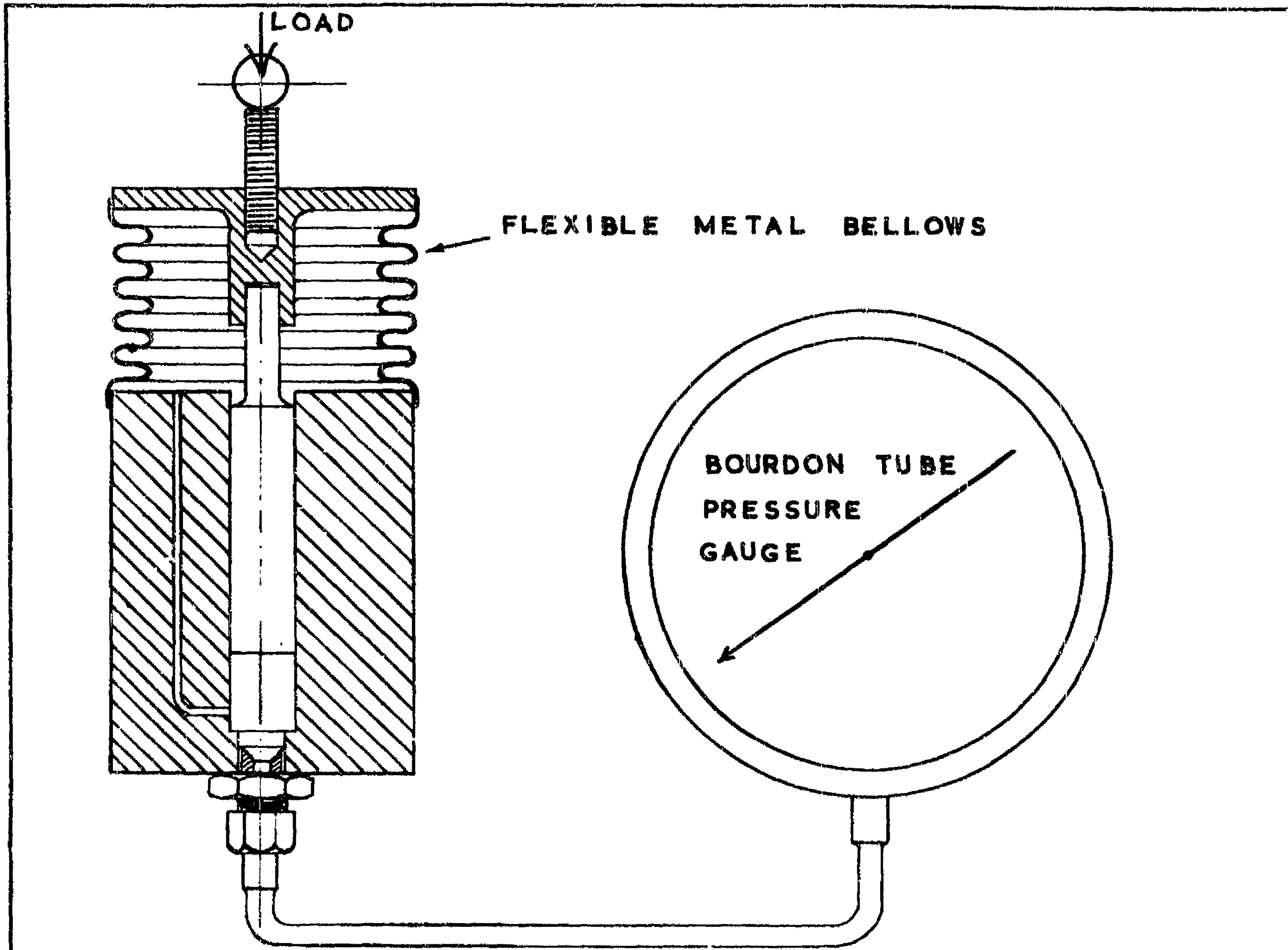
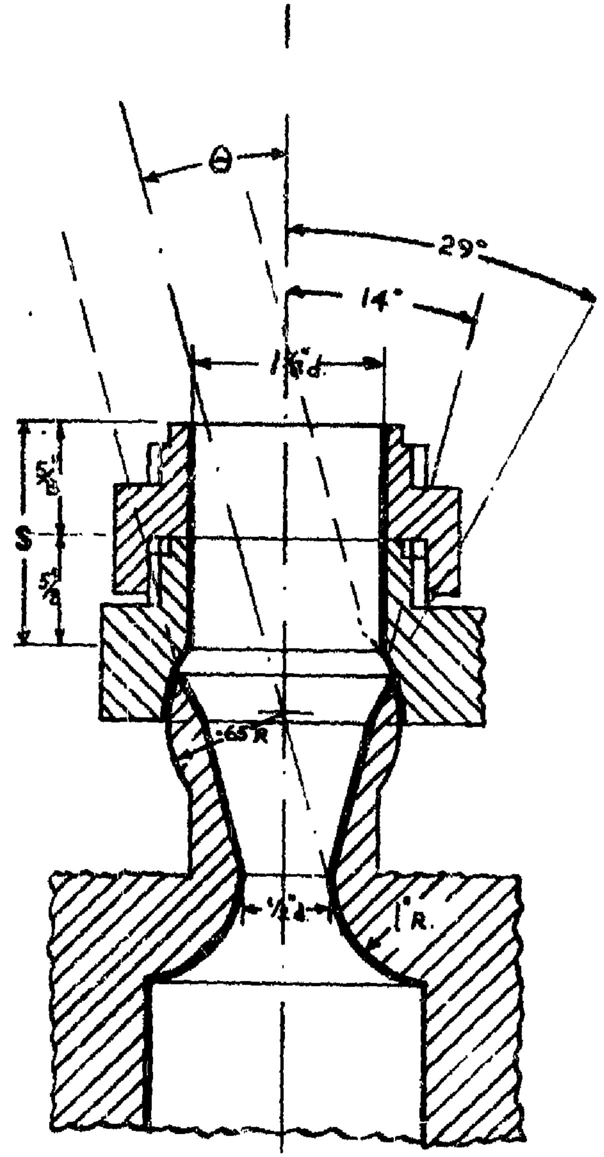
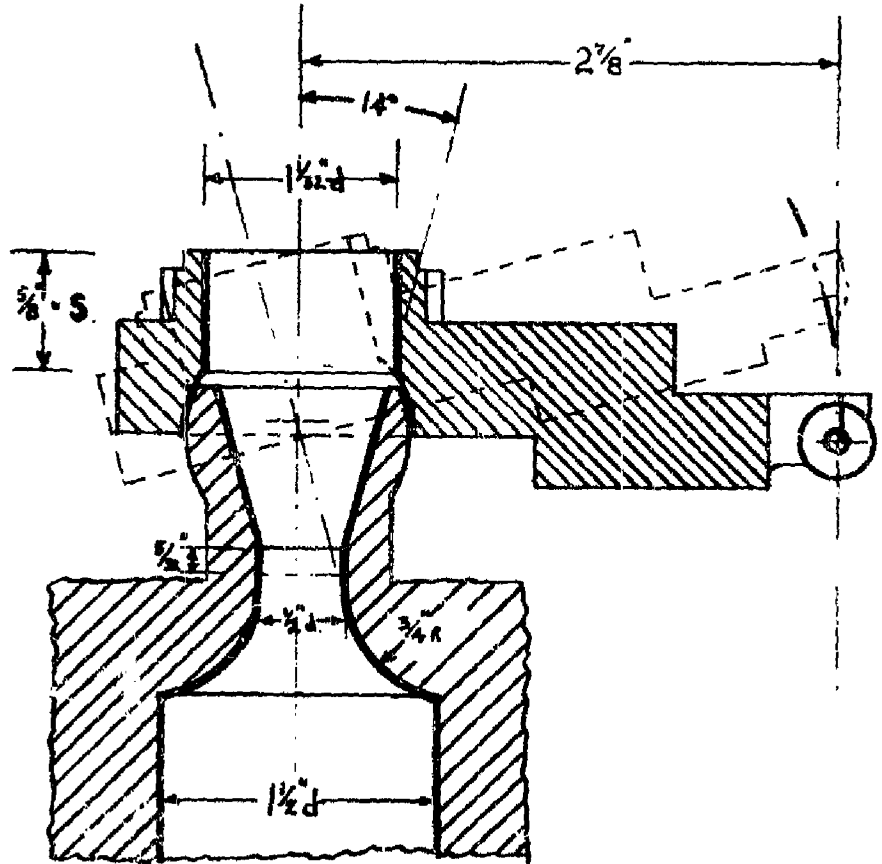


FIG. 4.

	TYPE a	TYPE b
THROAT DIAM.	·5012"	·4995"
THROAT AREA	·001369	·001360 ft²
EXIT AREA	·00769	·00480 ft²
EXT'N LENGTHS	5/8" + ·002"	5/8" − ·005"



TYPE a

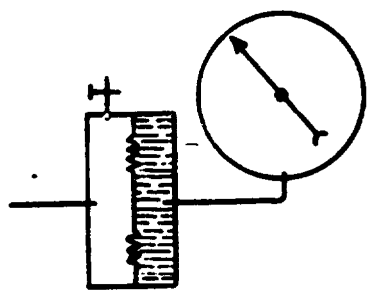


TYPE b

FIG. 5.

PHOTOGRAPH OF GAUGES

PLATE 6



DIAPHRAGM SEALED GAUGE

FIG. 7.

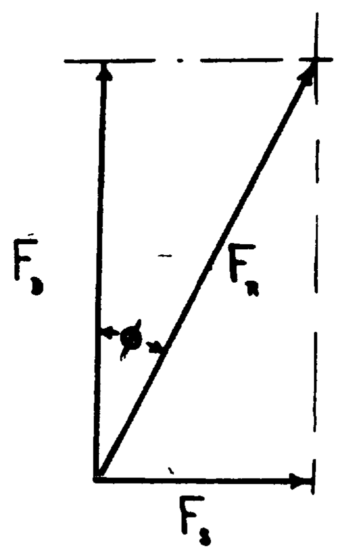
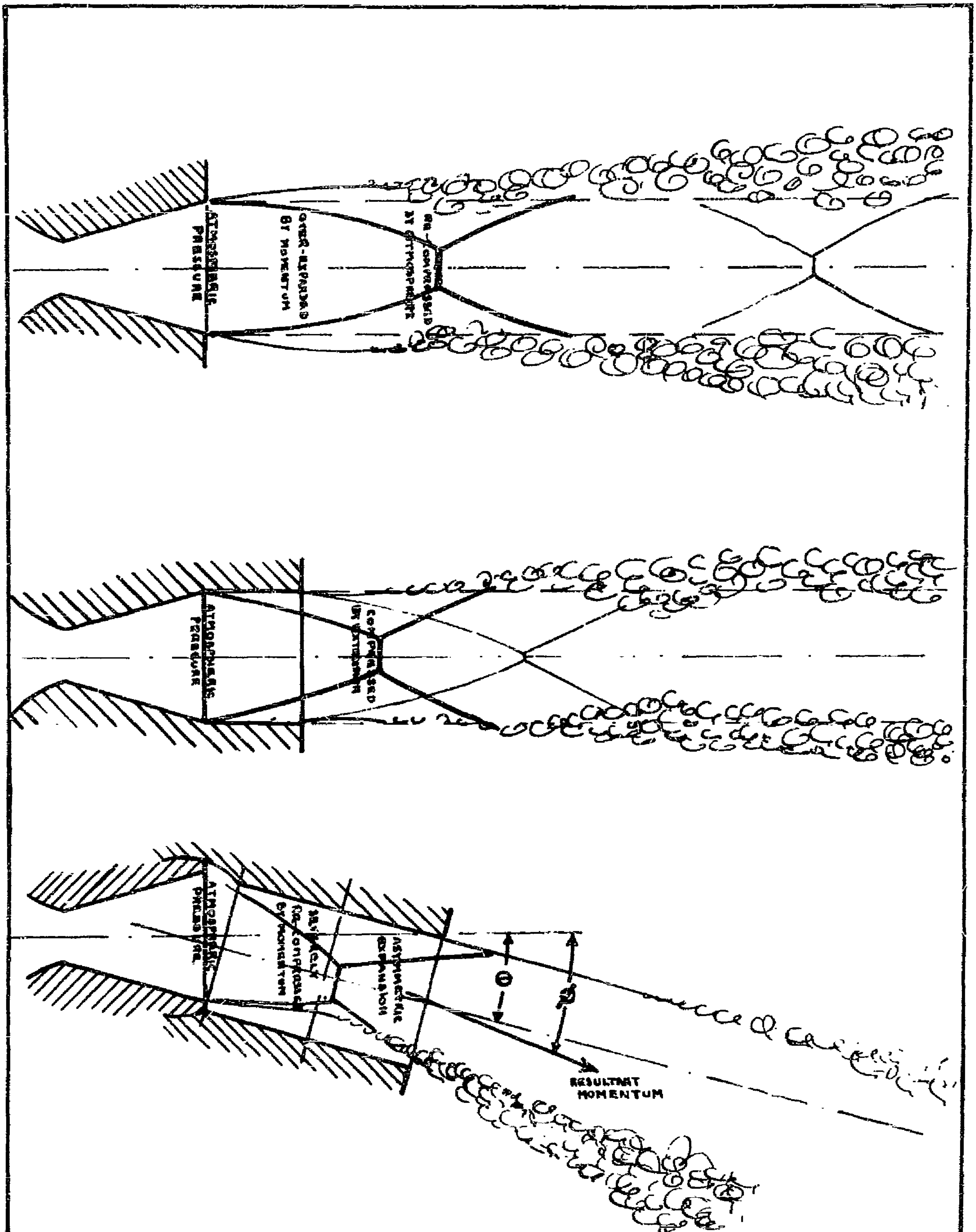


FIG. 13a.



FIGS. 14 - 16.

AIRFLOW THROUGH NOZZLE AND TUBE

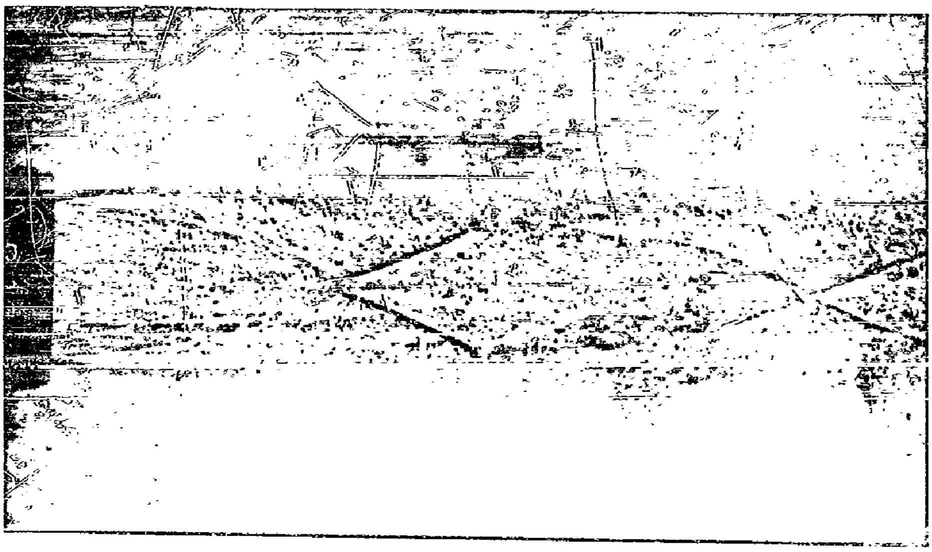


PLATE 17. p_o = 400 LBS/INS^2 TYPE "b" - PLAIN NOZZLE.

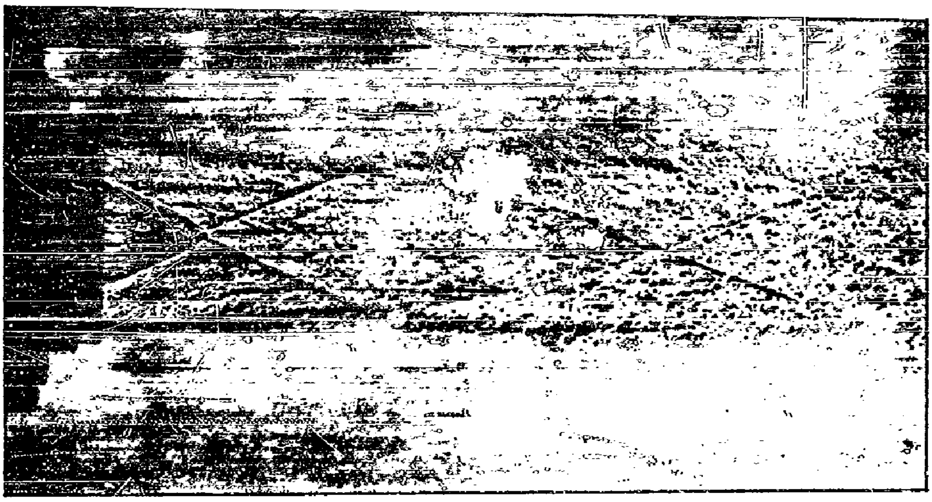


PLATE 18. p_o = 400 LBS/INS^2 TYPE "b" - $S = \frac{5}{8}$" $\theta = 0°$

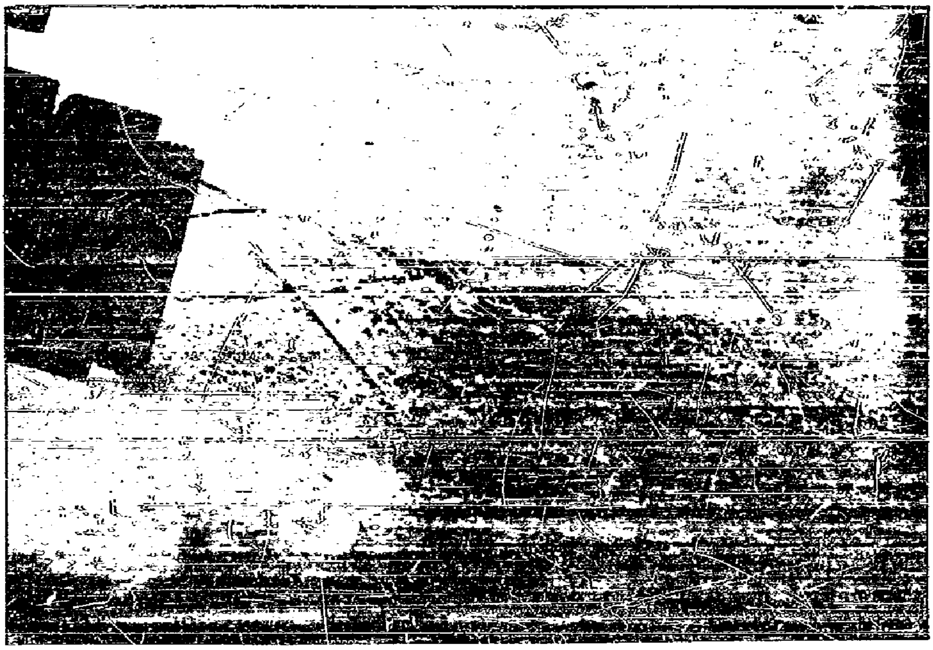


PLATE 19. p_o = 400 LBS/INS^2 TYPE "b" - $S = 1\frac{7}{8}$" $\phi = 18°$ $\theta = 15°$

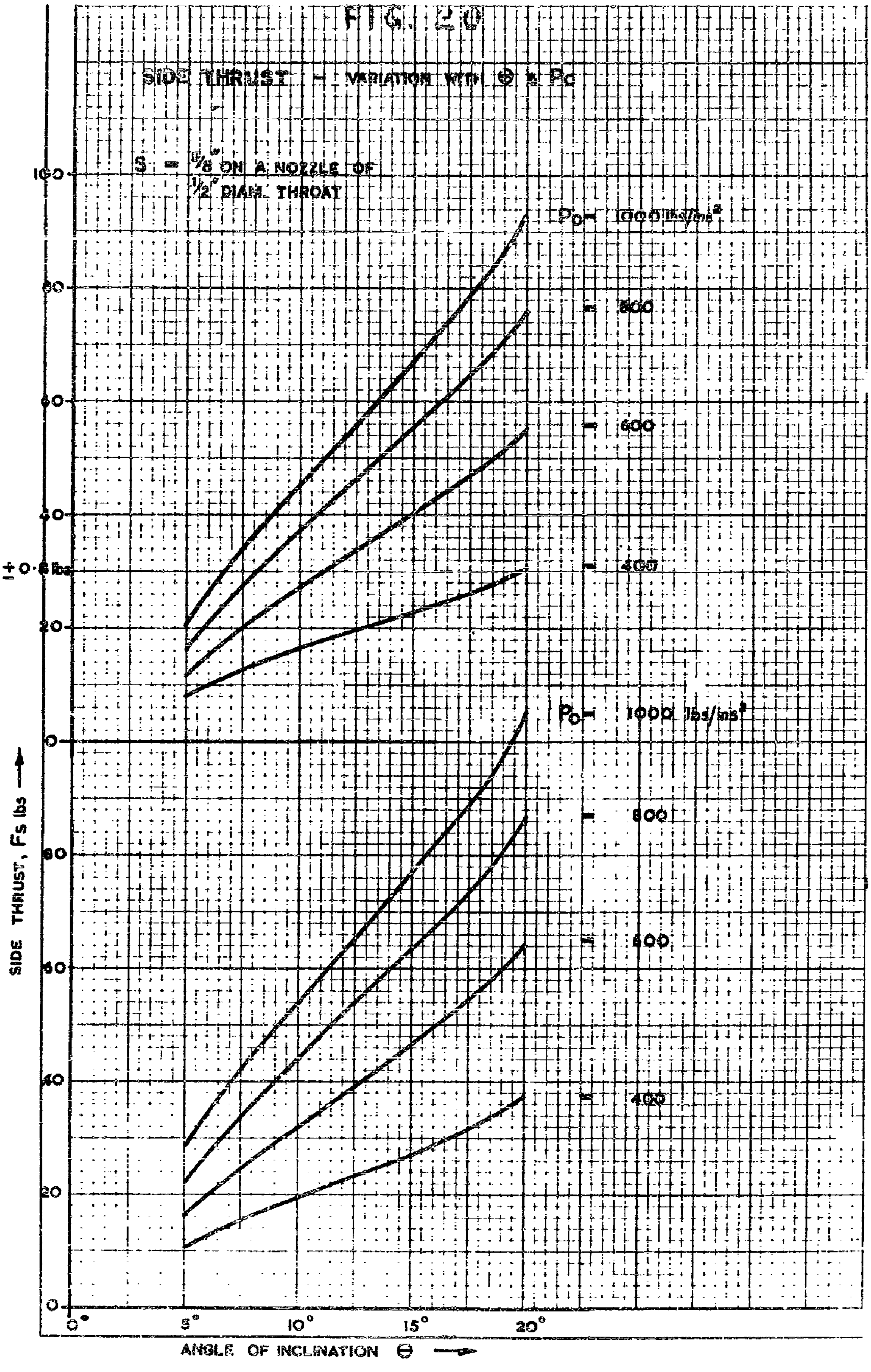


FIG. 20
SIDE THRUST – VARIATION WITH Θ & Pc
S = 5/8" ON A NOZZLE OF 1/2" DIAM. THROAT
Pc = 1000 lbs/ins²
= 800
= 600
= 400
Pc = 1000 lbs/ins²
= 800
= 600
= 400
100
80
60
40
1 + 0·8 lbs
20
0
80
60
40
20
0
SIDE THRUST, Fs lbs
0°
5°
10°
15°
20°
ANGLE OF INCLINATION Θ

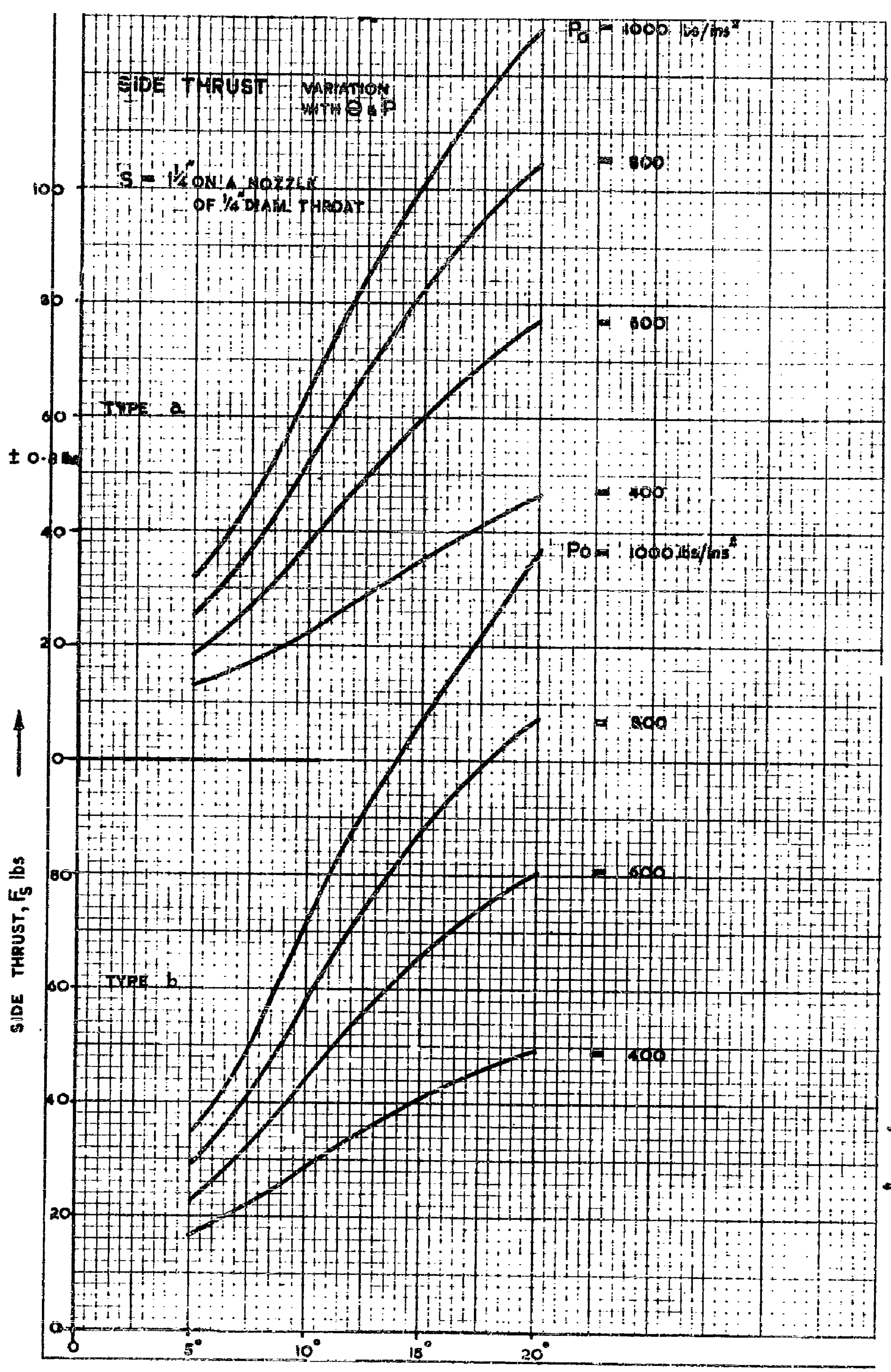
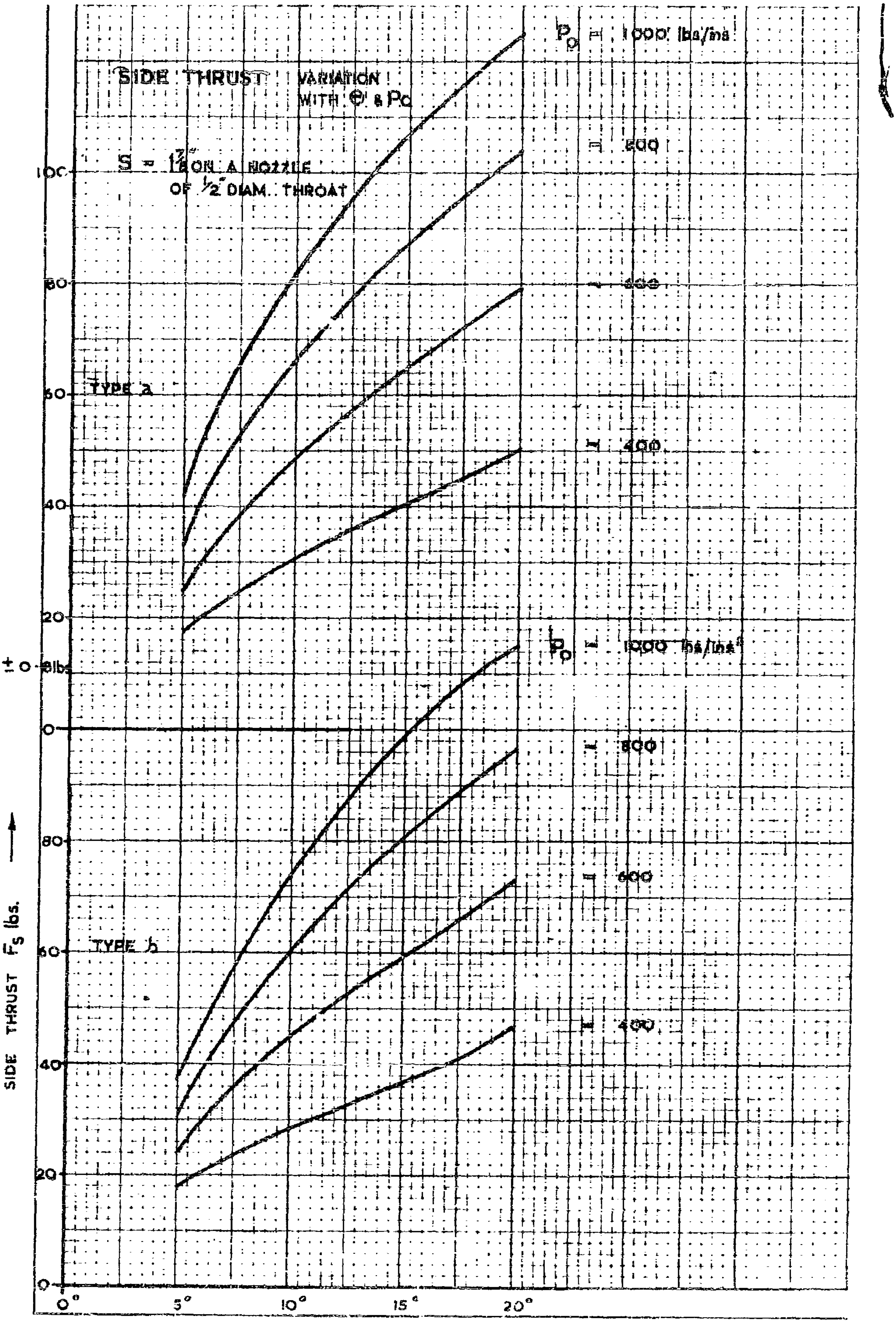

SIDE THRUST VARIATION WITH θ & P
S = 1/4" ON A NOZZLE OF 1/4" DIAM. THROAT
TYPE a.
TYPE b
± 0·8 lbs
P_O = 1000 lbs/ins²
= 800
= 600
= 400
Po = 1000 lbs/ins²
= 800
= 600
= 400
100
80
60
40
20
0
80
60
40
20
0
0
5°
10°
15°
20°
SIDE THRUST, F_S lbs
ANGLE OF INCLINATION θ

FIG. 21.



SIDE THRUST VARIATION WITH Θ & Po
S = 1 7/8" ON A NOZZLE OF 1/2" DIAM. THROAT
Po = 1000 lbs/ins
= 800
= 600
= 400
TYPE a
Po = 1000 lbs/ins²
= 800
= 600
= 400
TYPE b
100
80
60
40
20
1+0 lbs
0
80
60
40
20
0
SIDE THRUST Fs lbs.
0°
5°
10°
15°
20°
ANGLE OF INCLINATION Θ

FIG. 22.

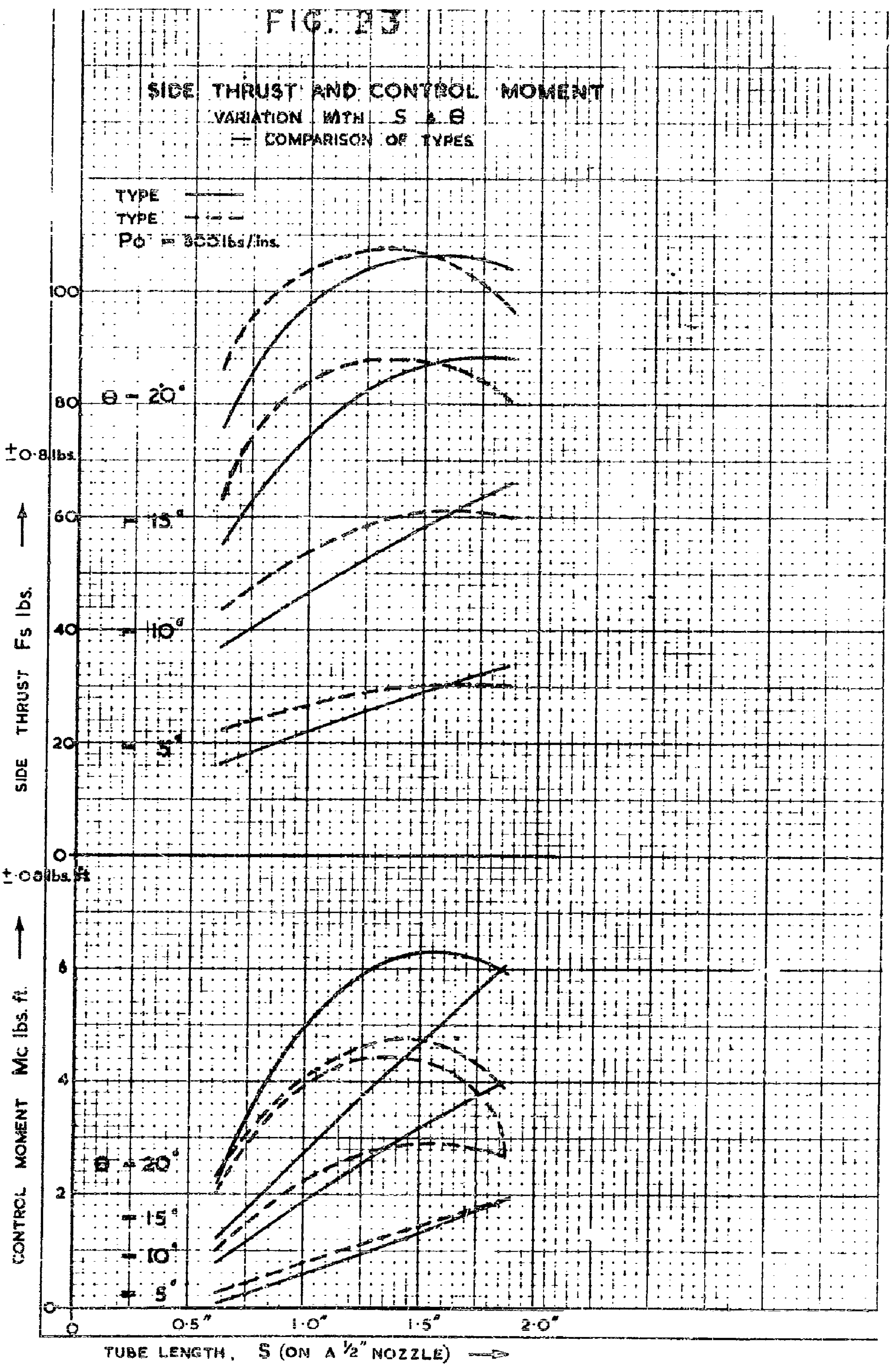


FIG. E3
SIDE THRUST AND CONTROL MOMENT
VARIATION WITH S & θ
— COMPARISON OF TYPES
TYPE
TYPE
Po = 300 lbs/ins.
θ = 20°
= 15°
= 10°
= 5°
θ = 20°
= 15°
= 10°
= 5°
100
80
60
40
20
0
6
4
2
0
±0·8 lbs.
±0·08 lbs. ft
SIDE THRUST Fs lbs.
CONTROL MOMENT Mc lbs. ft.
0
0·5"
1·0"
1·5"
2·0"
TUBE LENGTH, S (ON A ½" NOZZLE)

FIG. 24

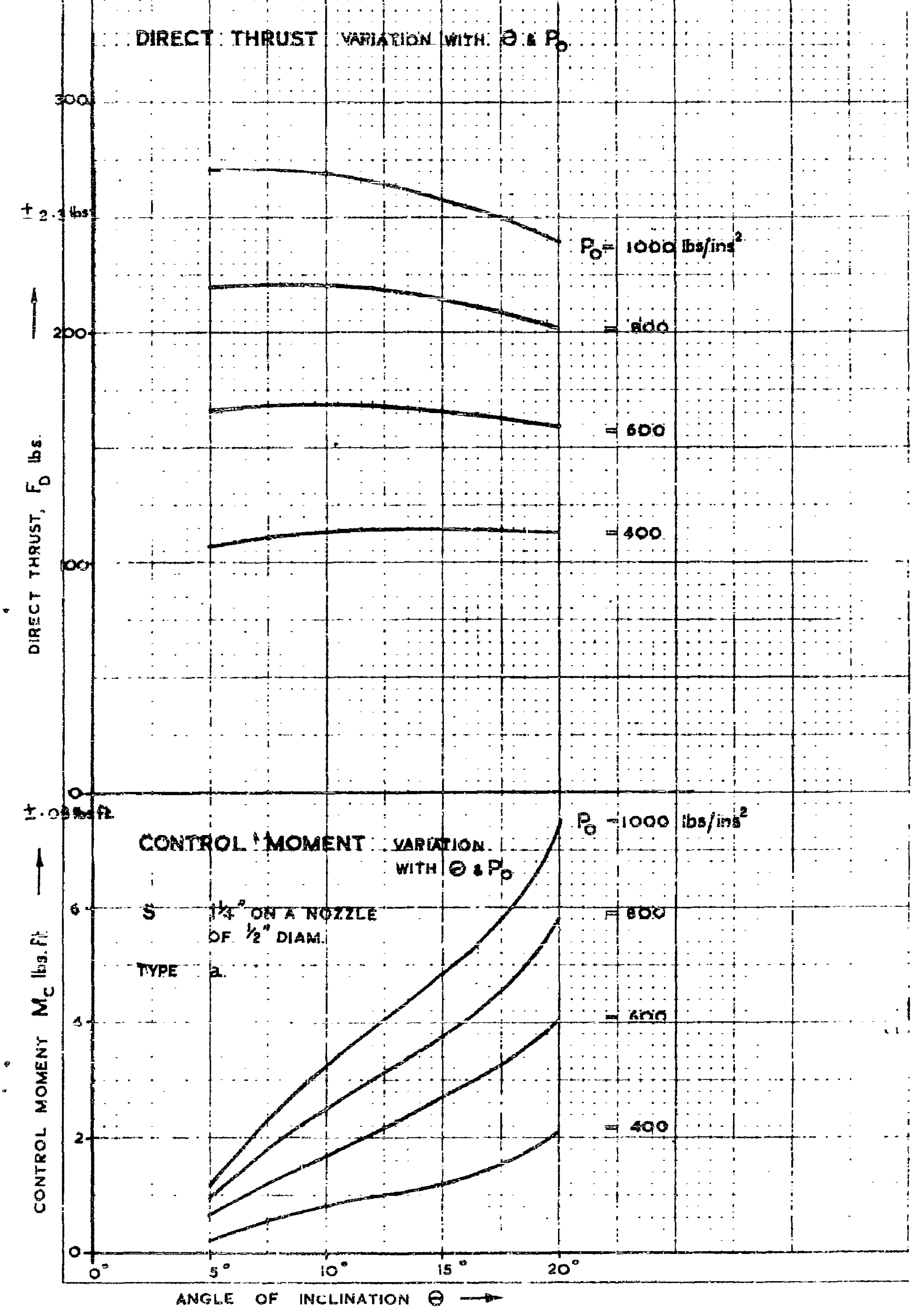

DIRECT THRUST VARIATION WITH Θ & P_o
300
± 2.3 lbs
200
100
0
P_o = 1000 lbs/ins²
= 800
= 600
= 400
DIRECT THRUST, F_D lbs.
± ·08 lbs ft.
CONTROL MOMENT VARIATION WITH Θ & P_o
S 1¼" ON A NOZZLE OF ½" DIAM.
TYPE a.
6
4
2
0
P_o = 1000 lbs/ins²
= 800
= 600
= 400
CONTROL MOMENT M_c lbs. ft.
0°
5°
10°
15°
20°
ANGLE OF INCLINATION Θ

FIG. 25

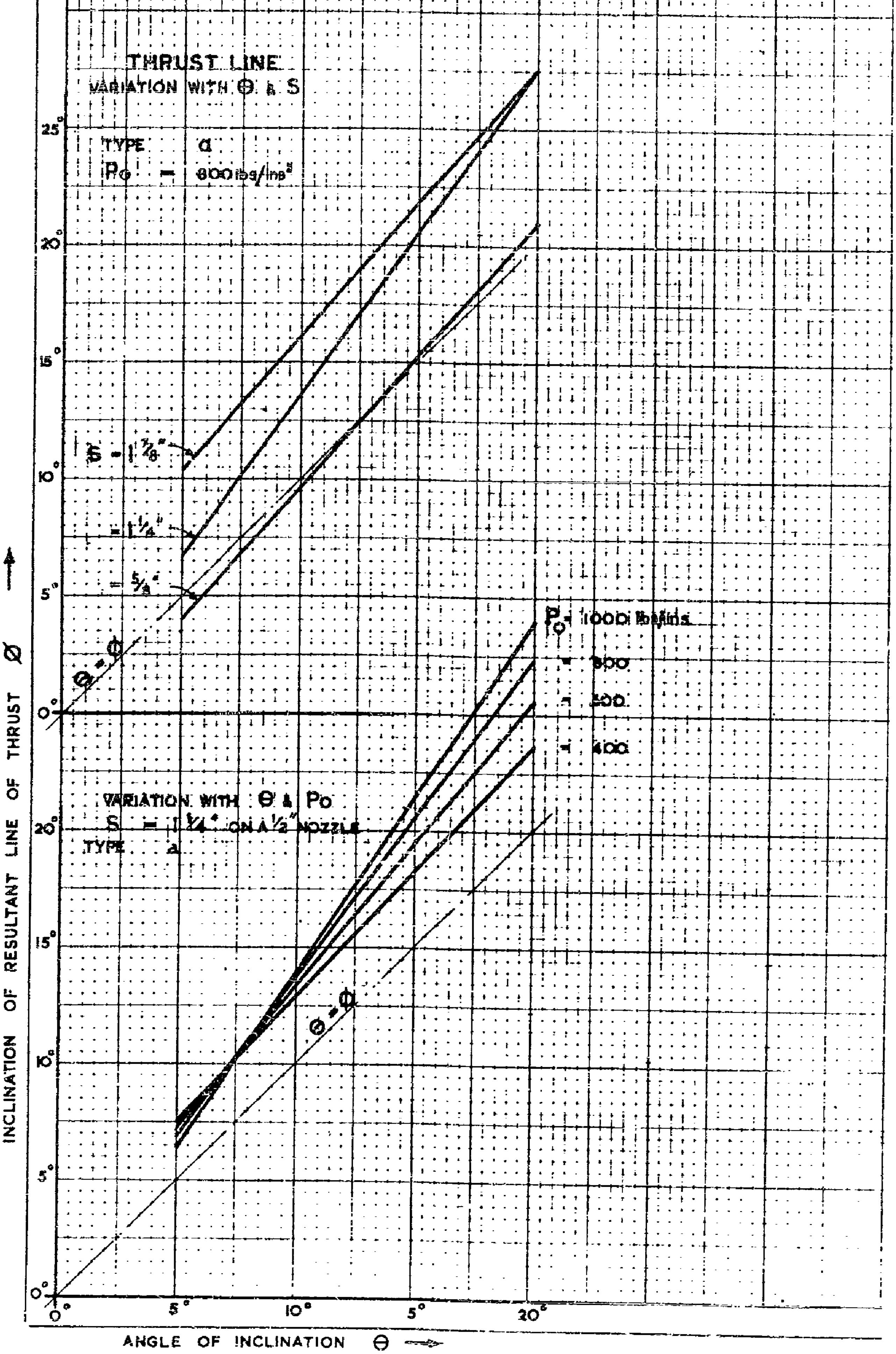
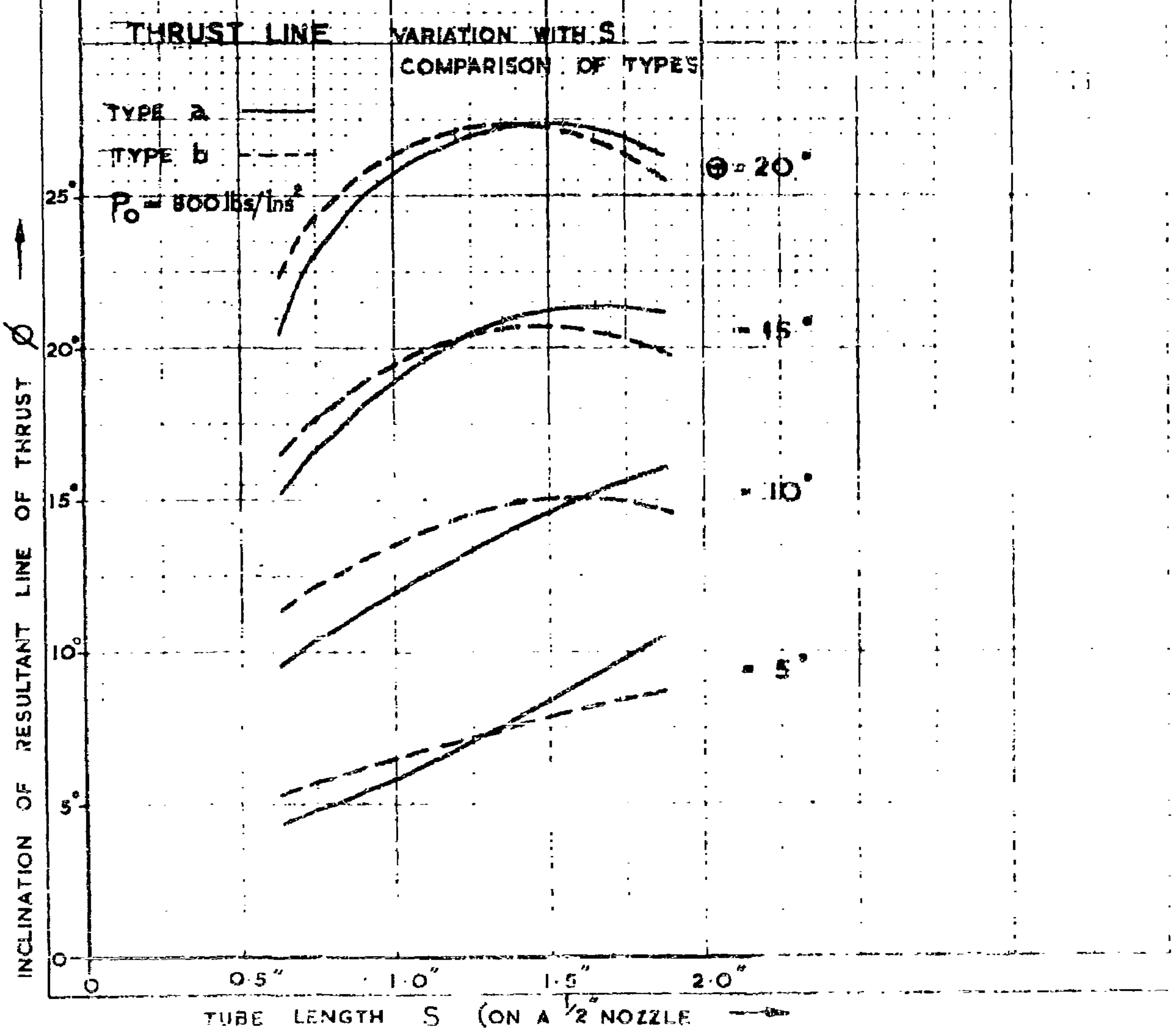

THRUST LINE
VARIATION WITH Θ & S
TYPE a
Po = 800 lbs/ins²
S = 1 7/8"
= 1 1/4"
= 5/8"
Θ = Φ
Po = 1000 lbs/ins
= 400
VARIATION WITH Θ & Po
S = 1 1/4" ON A 1/2" NOZZLE
TYPE a
Θ = Φ
INCLINATION OF RESULTANT LINE OF THRUST Ø
ANGLE OF INCLINATION Θ

FIG. 26


THRUST LINE VARIATION WITH S
COMPARISON OF TYPES
TYPE a
TYPE b
Po = 800 lbs/ins²
Θ = 20°
= 15°
= 10°
= 5°
INCLINATION OF RESULTANT LINE OF THRUST Φ
25°
20°
15°
10°
5°
0
0
0.5"
1.0"
1.5"
2.0"
TUBE LENGTH S (ON A ½" NOZZLE

AD- [redacted]

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